PART I

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. If $0 < x < 1$, then $1 + x^2 [\{x \cos (\cot^{-1} x) + \sin (\cot^{-1} x)\}^2 - 1]^{\frac{1}{2}} =$

   (A) $\frac{x}{1 + x^2}$ (B) x (C) $x\,1 + x^2$ (D) $1 + x^2$

Sol. (C)

The given expression $1 + x^2 \left[\left\{ x \times \frac{x}{1+x^2} + \frac{1}{1+x^2} \right\}^2 - 1\right]^{\frac{1}{2}}$

$= 1 + x^2 \left[\frac{(x^2 + 1)^2}{x^2 + 1} - 1\right]^{\frac{1}{2}}$

$= 1 + x^2 \left[x^2 + 1 - 1\right]^{\frac{1}{2}} = x\,1 + x^2 .$

ÆÆÆ

2. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors a, b, c such that $a \cdot b = b \cdot c = c \cdot a = \frac{1}{2}$.

   ÆÆÆÆÆÆÆÆÆ .

   Then, the volume of the parallelopiped is

   (A) $\frac{1}{2}$ (B) $\frac{1}{2\,2}$ (C) $\frac{3}{}$ (D) $\frac{1}{3}$

2
Sol. (A)
r 3 Æ r 1Æ
1 1 Æ 2Æ
Qa = Æb = Æ+ j, c = i + j+
i, Æ i k
2 2 2 2 3 3

1 0 0
1 3
0 1
2 2 =
Volume =
2
1 1 2
2 2 3 3

3. Consider the two curves

$C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 \quad 6x + 1 = 0$

Then,

(A) C1 and C2 touch each other only at one point

(B) C1 and C2 touch each other exactly at two points

(C) C1 and C2 intersect (but do not touch) at exactly two points

(D) C1 and C2 neither intersect nor touch each other

Sol. (B)

2
∴ y = 4x ....(i) and x2 + y2 6x + 1 = 0...... (ii)

Solving (i) and (ii)

x2 + 4x 6x + 1 = 0⇒ x2 2x + 1 = 0

⇒ (x 1)2 = 0

⇒ x=1

∴ y = ±2.

∴ C1 and C2 touch each other exactly at two points.

4. The total number of local maxima and local minima of the function f(x) = $\begin{cases} (2 + x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x \le 2 \end{cases}$

is

(A) 0 (B) 1 (C) 2 (D) 3

Sol. (C)

The graph of the function is

3
0 2
2 1

There is one local maxima and one local minima.

5. Let a and b non-zero real numbers. Then, the equation $(a x^2 + by^2 + c)(x^2 \quad 5xy + 6y^2) = 0$ represents

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Sol. (B) $x^2 - 5xy + 6y^2 = 0$

$\Rightarrow x^2 - 3xy - 2xy + 6y^2 = 0$

$\Rightarrow x(x - 3y) - 2y(x - 3y) = 0$

$\Rightarrow (x - 3y)(x - 2y) = 0 \Rightarrow$ two straight lines.

and when a = b and sign of c is opposite of a the equation $ax^2 + by^2 + c = 0$ represent circle.

(x – 1)n

6. Let g(x) = ;0 < x < 2, m and n are integers, m ≠ 0, n > 0, and let p be the left hand

logcosm (x – 1)

derivative of x – 1 at x = 1.

If lim g(x) = p, then

x →1+

(A) n = 1, m = 1 (B) n = 1, m = 1

(C) n = 2, m = 2 (C) n > 2, m = n

Sol. (C)

Q p = 1,

n

(x – 1)

Lt

+ g(x) = –1 ⇒ = –1

Lt m

x →1 + log cos (x −1)

x→1

n–1

n(x – 1)

Lt

⇒ = –1

\+ m –1

1

x →1 × m cos (x −1) × (−sin(x −1)

m

cos (x – 1)

n–2

n(x – 1) · (x – 1)

Lt n– 2

n (x – 1)

⇒ = –1.

\+

⇒ Lt = –1

1

x → 1 −sin (x −1) × ×m
x →1+ m
cos (x – 1)

Limit to be exist n – 2 = 0 ⇒ n = 2. and m = 2

SECTION - II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B),
(C) and (D), out of which ONE OR MORE is/are correct.

n−1
n n
∑ n2 + kn + k 2 , for n = 1, 2, 3, K. Then,
n
∑ n2 + kn + k 2 and Tn =
Let Sn =
7.
k =0
k =1

(A) Sn < π / 3 3 (B) Sn > π / 3 3

(C) Tn < π / 3 3 (D) Tn > π / 3 3

Sol.
n
n
∑
Q Sn =
2
2
⎛ n⎞
k =1 3n
+⎜k + ⎟
⎝ 2⎠
4
n
1 1
∑n
=
2
3 ⎛1 k⎞
k =1
+ +
4⎜2 n⎟
⎝ ⎠
1 n 1
∑
S∞ = Lt Sn = Lt
2

$$\lim_{n\to\infty} \qquad \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{1}{\frac{3}{4} + \left(\frac{1}{2}+\frac{k}{n}\right)^2}$$

$$= \int_0^1 \frac{dx}{\frac{3}{4} + \left(\frac{1}{2}+x\right)^2}$$

$$= \left[\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{\frac{1}{2}+x}{\frac{\sqrt{3}}{2}}\right]_0^1 = \frac{\pi}{3\sqrt{3}}$$

Similarly we can calculate T_∞

∵ $S_1 < S_2 < S_3 < \ldots S_n < \ldots S_\infty$ and $T_1 > T_2 > T_3 > \ldots T_n > \ldots T_\infty$

$\Rightarrow S_n < \frac{\pi}{3\sqrt{3}}$ and $T_n > \frac{\pi}{3\sqrt{3}}$.

8. Let f(x) be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'\left(\frac{1}{4}\right) = 0$. Then,

(A) $f''(x)$ vanishes at least twice on [0,1]

(B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx = 0$

(D) $\int_0^{1/2} f(t)\,e^{\sin \pi t}\,dt = \int_{1/2}^{1} f(1-t)\,e^{\sin \pi t}\,dt$

Sol. (A, B, C, D)

(a, b, c, d)

f(x) = f(1 x)

f '(x) = f (1 x)

At $x = \frac{1}{2}$, $f'\left(\frac{1}{2}\right) = f'\left(\frac{1}{2}\right) \Rightarrow f'\left(\frac{1}{2}\right) = 0$

At $x = \frac{1}{4}$, $f'\left(\frac{1}{4}\right) = f'\left(\frac{3}{4}\right) \Rightarrow f'\left(\frac{3}{4}\right) = 0$

$\therefore f'\left(\frac{1}{4}\right) = f'\left(\frac{1}{2}\right) = f'\left(\frac{3}{4}\right) = 0$

$\therefore$ By Rolle s theorem, there exists.

c_1, c_2 between $\left(\frac{1}{4}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{3}{4}\right)$ respectively.

Such that $f''(c_1) = f''(c_2) = 0$

$$\int_{1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\, dx$$

$$= \int_{1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin(x)\, dx$$

$$= \int_{1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\, dx$$

$\left(\text{as } f(x) = f(1 \quad x) \text{ let } x = x + \frac{1}{2} \Rightarrow f\left(x + \frac{1}{2}\right) = f\left(\frac{1}{2} \quad x\right)\right)$

$$\Rightarrow \int_{1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\, dx = 0$$

$$\int^{1/2} f(t)\, e\, dt$$

sin πt

let t = 1 − x

0

dt = − dx

1/ 2

sin π(1 − x)

∫f (1 − x) e dx

1

1

∫f (1 − x) e dx

sin πx

=

1/ 2

9. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcirle, then

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{QS \times SR}$ (B) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{QS \times SR}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (D) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

Sol. (B,D)

∵ $PS \times ST = QS \times SR$

Apply GM > HM for PS and ST

$$PS \times ST > \frac{2}{\frac{1}{PS} + \frac{1}{ST}}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{QS \times SR}$$

and apply AM > GM on QS, SR

$$\frac{QR + SR}{2} > QS \times SR$$

$$\frac{QR}{2} > QS \times SR$$

$$\frac{1}{QS \times SR} > \frac{2}{QR}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{QS \times SR} > \frac{4}{QR}$$

Let P(x1, y1) and Q (x2, y2), y1 < 0, y2 < 0, be the end points of the latus rectum of the ellipse x2 +
10.
4y2 = 4. The equations of parabolas with latus rectum PQ are
(A) x 2 + 2 3 y = 3 + 3
(B) x 2 − 2 3 y = 3 + 3
(C) x 2 + 2 3 y = 3 − 3
(D) x 2 − 2 3 y = 3 − 3

Sol. (B, C)

2 2
x y
+ =1
4 1

(0,1)
(√ 3,0) (√ 3,0)
(2,0)
(2,0)
P (√3, 1/2)
Q (√ 3 , 1 /2)
(0, 1)

Focus ⎛ −1⎞
⎜ 0, ⎟
⎝ 2 ⎠

−1
Directrix y = ± 3
2

∴ Equation of parabola becomes

2 2
(x − 0)2 + ⎛ y +
1⎞ ⎛ ⎞
1
⎟= ⎜ y + 2 m 3 ⎟
⎜
⎝ 2⎠ ⎝ ⎠
2
⎛ 1⎞ ⎛ 1⎞
= ⎜ y + ⎟+ 3 ± 2 3⎜ y + ⎟
⎝ 2⎠ ⎝ 2⎠
x = 3 ± 3 (2y + 1)
2
2
x m 2 3y = 3 ± 3
ie. x 2 − 2 3y = 3 + 3
or x 2 + 2 3y = 3 − 3

SECTION - III
Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

Let f and g be real valued functions defined on interval (1, 1) such that g'' (x) is continuous, 11.

$g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

STATEMENT - 1 : $\lim_{x \to 0} [g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$.

and

STATEMENT - 2 : $f'(0) = g(0)$.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (B)

$f(x) = g(x) \sin x$

$f'(x) = g(x)\cos x + g'(x)\sin x$

$f'(0) = g(0)$ ....(II)

$f''(x) = -g(x) \sin x + g'(x)\cos x + g'(x) \cos x + g''(x) \sin x$

$= -g(x) \sin x + 2g'(x)\cos x + g''(x) \sin x$

$f''(0) = 0$

$\lim_{x\to 0} \frac{g(x) \cos x - g(0)}{\sin x}$ $\left(\frac{0}{0} \text{ form}\right)$

$\lim_{x\to 0} \frac{g'(x) \cos x - g(x) \sin x}{\cos x} = 0 = f''(0)$

12. Consider three planes

P1 : x − y + z = 1

P2 : x + y − z = −1

P3 : x − 3y + 3z = 2.

Let L1, L2, L3 be the lines of intersection of the planes P2 and P3, P3 and P1, and P1 and P2, respectively.

STATEMENT - 1 : At least two of the lines L1, L2 and L3 are non-parallel.

and

STATEMENT - 2: The three planes do not have a common point.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 s True, STATEMENT - 2 True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 s False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (D)

Gives L1, L2, L3 have direction ratios 0 :1 :1

all are parallel

Here D = 0 = D1

D2 = D3 = 2

No solution

13. Consider the system of equations

ax + by = 0, cx + dy = 0, where a,b,c,d ∈ {0,1}.

STATEMENT - 1 : The probability that the system of equation has a unique solution is $\frac{3}{8}$.

and

STATEMENT - 2 : The probability that the system of equations has a solution is 1.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(C) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (B)

There are 16 determinats of entry 0 and 1.

and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ are six non-zero determinats.

∴ Probability that system has unique solution is $\frac{6}{16} = \frac{3}{8}$.

⇒ Statement 1 is correct.

and Q It is a homogeneous equation

⇒ System has a solution ⇒ Probability is 1.

14. Consider the system of equations

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1.$$

STATEMENT - 1 : The system of equation has no solution for k ≠ 3.

and

STATEMENT - 2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for k ≠ 3.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (A)

1 2 3
Q D = 1 1 2 = 0
1 3 4

−1 2 3
D1 = k 1 2
1 3 4

= (4 6) + 2(4k + 2) + 3(3k 1)

= 2 + 8k + 4 9k 3

= k + 3 ≠ 0, k ≠ 3.

⇒ System has no solution for K ≠ 3

⇒ Statement 1 is correct.

Again

1 3 1
D2 = 1 2 K
1 4 1

⇒ D2 = 1(2 4K) 3(1 K) 1(4 + 2)

= 2 4K + 3 + 3K + 4 2

= K + 3 ≠ 0,k ≠ 0

⇒ Statement 2 is true and correct explanation of statement 1

Section IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be
answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Paragraph for question no. 15 to 17.

Let A, B, C be three sets of complex numbers as defined below.

A = {z : Im z ≥ 1}

B = {z : z − 2 − i = 3}

C = { z : Re (1 − i) z = 2 }

15. The number of elements in the set A ∩ B ∩ C is

(A) 0 (B) 1 (C) 2 (D) ∞

Sol. (B)

To answer this question, are needs to draw the area/region as marked by A, B & C

For A = {z : Im(z) ≥ 1}

A denotes area/region (in Argand plane) beyond y ≥ 1

For B = {z : z 2 i = 3}

B denotes point on the circle (in Argand plane) with centre at (2, 1) and radius of 3 units.

For C = { z : R (1 i) z = 2 }

C denotes points on the line x + y = 2

y
P
(0,1)
(2,1)
y=1
x

Hence, A ∩ B ∩ C will be only one point P which is intersection of line x + y = 2 and circle :
(x 2)2 + (y 1)2 = 9.

16. Let z be any point in A ∩ B ∩ C . Then, z + 1 − i 2 + z − 5 − i 2 lies between

(A) 25 and 29 (B) 30 and 34 (C) 35 and 39 (D) 40 and 44

Sol. (C)

z is the point P as per solution of question 15.

2 2

Now, if we look carefully at z + 1 – i + z – 5 – i , we can see that it is nothing but sum of square of distance between P and Q(1, 1) and P and R(5, 1).

P

R (5,1)

(1,1)

Q

As is evident that Q and R are on circle itself at two ends of diameters.

Hence,

PQ2 + PR 2 = QR 2

As QR = 6

∴ QR2 = 36 = PQ2 + QR2

17. Let z be any point in A ∩ B ∩ C and let w be any point satisfying ω − 2 − i < 3. Then, z − ω + 3

lies between

(A) 6 and 3 (B) 3 and 6 (C) 6 and 6 (D) 3 and 9

Sol. (B)

Now ω is defined by ω − 2 − i < 3 ,

Which means ω is all the points inside circle represented by (x − 2) + (y − 1) = 9 in the Argand

2 2

plane.

Now, we have to find out coordinates of point P. to find the range of z – ω + 3

To find coordinates of point P we have to solve the following equations simultaneously.

x+y = 2

(x – 2)2 + (y – 1)2 = 9

on solving, we get

( ) ( )

2x 2 – 2 1 + 2 x – 2 1 + 2 = 0

( ) ( )

x2 – 2 1 + 2 x – 2 1 + 2 = 0

(1 + 2) – 7+6 2

x= (+ve, sign to be ignored as P is in second quadrant)

2

2.4 – 15.4

x= = –0.76

2

as x + y = 2 ⇒ y = 2.16

P ≡(−0.76, 2.16)

(0.76)2 + (2.16)2

z = = 0.58 + 4.67 = 5.25 = 2.3

z + 3 = 5.3

Range of ω can be found by finding OM and ON

M

Q (2,1)

O

N

OM = distance before O(0, 0) to centre (2, 1) + Radius of circle denoted by (x − 2) + (y − 1) = 9

2 2

∴ OM = 4 + 1 + 3 = 5 + 3

∴ OM = 5.3

Further ON = MN OM = 2×3 3

∴ ON = 0.7

Hence 0.7 < ω < 5.3

Paragraph for question no. 18 to 20

Consider the functions defined implicitly by the equation y3 − 3y + x = 0 on various intervals in the real
line.

If x ∈ (−∞, −2) ∪ (2, ∞), the equation implicitly defines a unique real valued differentiable function y = f(x).

If x ∈ (−2,2), the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying
g(0) = 0.

( ) ( )

If f −10 2 = 2 2, then f '' −10 2

18. =

4 2 4 2 4 2 4 2

(B) − (D) −

(C)

(A) 3 2 3 2 3

73 3

7 3 7 3 7 3

Sol. (A)

y3 − 3y + x = 0

1

∴ 3y 2 y '− 3y '+ 1 = 0 , y' =

( )

3 1 − y2

⇒ 3y '' y 2 − 3y ''+ 6y (y ') = 0

2

( )

⇒ y '' y 2 − 1 = −2y (y ')

2

2y 1 2y

⇒ y'' = =

(1 − y) ⎡⎣3 (1 − y2)⎤⎦ ( )

2 3

2

9 1 − y2

( )

Q f −10 2 = 2 2

( ) 2.2 2

∴ f " −10 2 = 4 2

3

=

( )

⎛ 2⎞

9⎜1 − 2 2 73 .32

⎟

⎝ ⎠

19. The area of the region bounded by the curve y = f(x), the x-axis and the lines x = a and x = b, where

−∞ < a < b < −2 is

b

b x

x

(B) − ∫ dx + b f(b) − af(a)

∫

((f (x))2 − 1)

dx + b f(b) − af(a)

((f (x)) − 1)

(A)

2

a3

a3

b b

x x

∫ (D) −∫

dx − b f(b) + af(a) dx − b f(b) + af(a)

( ) ((f (x)) − 1)

(C)

a 3 (f (x)) − 1

2 2

a3

Sol. (A)

Q ∫f (x) dx = x f (x) −∫x f '(x) dx

b

∫f(x) dx

∴ Required area =

a

b

x

=⎡ x f (x)⎤ − ∫

b

)

(

dx

⎣ ⎦a

a 3 1− f (x)

2

b

x

$\int$

= dx + b f(b) − a f(a)

((f (x))2 − 1)

a3

1

$\int$g'(x) dx =

20.

−1

(A) 2g(1)   (B) 0   (C)   2g(1)   (D) 2g(1)

Sol. (D)

1

$\int$g'(x) dx = ⌈g (x)⌉ −1

1

⌊   ⌋

−1

= g(1)   g(1)

= g(1)   (g(1))

= 2g(1)

Since given curve y3 − 3y + x = 0 is symmetric about origin

∴ if y = g(x)

⇒   y = g(x)

⇒ g(1) =   g(1)

Paragraph for question no. 21 to 23

21. A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the

3x + y − 6 and

sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation

⎛3 3 3⎞

the point D is ⎜ 2 , 2 ⎟. Further, it is given that the origin and the centre of C are on the same

⎝   ⎠

side of the line PQ.

2

(   )   ⎛   1⎞

(   )

2

2

+ (y − 1) = 1   (B) x − 2 3   +⎜ y +   ⎟=1

2

(A) x − 2 3

⎝   2⎠

(   )   (   )

2   2

+ (y + 1) = 1   + (y − 1) = 1

2   2

(C) x − 3   (D) x − 3

Sol. (D)

P

3 √3 , 3

D
F 2 2
1
2
G
(√3, 1)
R E Q
3x + y – 6 = 0

Equation of PQ
Radius of circle = 1

a
r= , a = length of side of equilateral triangle.
2 3

⇒a=2 3
Equation of DR is x – 3y + k = 0
3 3 3 3
⇒ – +k = 0 ⇒k = 0
2 2
∴ Equation of DR is x – 3y = 0
( )
∴ Co-ordinates of G ≡ 3, 1 as origin and centre of circle lie in the same side.
( )
2
+ (y – 1) = 1
2
∴ Equation of circle x – 3

22. Points E and F are given by

⎛ 3 3⎞ ⎛ 3 1⎞
( ) ( )
(A)⎜ 2 , 2 ⎟, (B)⎜ 2 , 2 ⎟,
3, 0 3, 0
⎜ ⎟ ⎜ ⎟
⎝ ⎠ ⎝ ⎠
⎛ 3 3⎞⎛ 3 1⎞ ⎛ 3 3 ⎞⎛ 3 1⎞
(C)⎜ 2 , 2 ⎟,⎜ 2 , 2 ⎟ (D)⎜ 2 , 2 ⎟,⎜ 2 , 2 ⎟
⎜ ⎟⎜ ⎟ ⎜ ⎟⎜ ⎟
⎝ ⎠⎝ ⎠ ⎝ ⎠⎝ ⎠

Sol. (A)

Q Slope of PQ is = – 3
1 3
∴ cos θ = – , sin θ =
2 2
⎛ 1⎞3 3
3 3 3
∴x = ± 3⎜– ⎟= m
⎝ 2⎠
2 2 2
= 3, 2 3
3 3
y= ± 3×
2 2

3 3
= ± = 3, 0
2 2

( ) ( )
∴ Co-ordinates of P and Q are 3, 3 and 2 3, 0

Co-ordinates of R is (h, k)

2×3 3
1× h +
2
∴ = 3
2 +1
h+3 3 = 3 3 ⇒ h = 0

3
1+ k × 2 ×
2 =1
and
1+ 2
k+3=3 ⇒k=0

∴ R = (0, 0)

Coordinates of E and F are

⎛ 3 +0 3+0⎞ ⎛2 3 +0 0+0⎞
⎜ ⎟and⎜ ⎟
, ,
⎜ 2 ⎟ ⎜ 2 ⎟
2 ⎠ 2
⎝ ⎝ ⎠

⎛ 3 3⎞
( )
i.e.⎜ 2 , 2 ⎟, 3, 0
⎜ ⎟
⎝ ⎠

23. Equations of the sides QR, RP are

2 2 1
(A) y = x + 1, y = – x –1 (B) y = – x, y = 0
3 3 3

3 3
(C) y = x + 1, y = – x –1 (D) y = 3x, y = 0
2 2

Sol. (D)

Equation of QR and PR are

y = 3x and y = 0

PHYSICS

PART II

SECTION I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

24. Figure shows three resistor configurations R1, R2 and R3 connected to 3V battery. If the power
dissipated by the configurations R1, R2 and R3 is P1, P2 and P3, respectively, then

Figure :

1Ω 1Ω
1Ω
1Ω
1Ω 1Ω
1Ω 1Ω
1Ω
3V 3V
3V
1Ω 1Ω
1Ω 1Ω
1Ω 1Ω
R2
R3
R1

(A) P1 > P2 > P3 (B) P1 > P3 > P2 (C) P2 > P1 > P3 (C) P3 > P2 > P1

Sol. (C)

After solving R1 = 1Ω

$$R2 = \frac{1}{2}\Omega$$

& R3 = 2Ω

& we know $P = \frac{V2}{R}$

P2 > P1 > P3

25. Which one of the following statements is WRONG in the context of X-rays generated from a X-ray
tube?

(A) Wavelengh of characteristic X-rays decreases when the atomic number of the target increases

(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target

(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-rays tube

(D) Cut-off wavelength of the continuous X-rays depends on he energy of the electrons in the X-rays tube

Sol. (B)

Theoritical

26. Student I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a
simple pendulum. They use different length of the pendulum and/or record time for different number
of oscillations. The observations area shown in the table .

Least count for length = 0.1 cm

Least count for time = 0.1 s

Length of the Number of Total time for (n)

Student	pendulum (cm)	oscillations (n)	oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If EI, EII and EIII are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students, I, II and II, respectively,

(A) EI = 0 (B) EI is minimum

(C) EI = EII (D) EII is maximum

Sol. (B)

We know

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2\frac{l}{g}$

$g = 4\pi^2\frac{l}{T^2} \Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$

Exp I

$\frac{\Delta g}{g}\times 100 = \left[\frac{.1}{64} + 2\times\frac{.1}{16}\right]\times 100$

$= \frac{[.1 + .2]\times 100}{64} = \left[\frac{30}{64}\right]$

because No. of oscillation taken by student I is 8 as compare to 4 of student II.

∴ Student I is more accurate as compare to II

So EI ≠ E2

Exp II:

$\frac{\Delta g}{g}\times 100 = \left[\frac{.1}{20} + 2\frac{.1}{9}\right]\times 100$

$= \frac{.9 + 4}{1000} = \frac{4.9}{1000}\times 100 = \frac{490}{1000}$

27. A spherically symmetric gravitational system of particles has a mass density

$\rho = \begin{cases} \rho_o & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$

⎩0

where ρo is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

V V

(B)

(A)

R R

r r

V V

(c) (d)

R R

r r

Sol. (C)

$$\rho = \begin{cases} \rho_o \dfrac{r}{R} & r \le R \\ 0 & r > R \end{cases}$$

Gravitational field inside the sphere of distance r from the centre

$$g = \frac{GM}{R^3} r$$

$$\text{where } M = \left[\frac{4}{3}\pi R^3 \rho_o\right]$$

$$\text{so } g = \frac{G\frac{4}{3}\pi R^3 \rho_o}{R^3} r$$

$$g = \frac{4}{3} G\pi\rho_o r$$

$$\therefore \text{ Inside the sphere } mg = \frac{mv^2}{r}$$

$$\Rightarrow g = \frac{v^2}{r}$$

$$\Rightarrow \frac{4}{3}\pi G\rho_o r^2 = v^2$$

$$\Rightarrow v = \sqrt{\frac{4}{3}\pi G\rho_o}\, r$$

$$\Rightarrow v \propto r$$

& for $r > R$

4 3

G× πR ρo
4 GπR3 ρo
GM 3
g= = =
r2 r2
r 3
so out side the sphere
mv 2 v2
mg = g=
,
r r

4 GπR3ρo v 2
=
r2
3 r
1
⇒v∝
r
V
so graph will be like this
R r

28. Two beams of red and violet colours are made to pass separately through a prism (angle of the
prism is 60°). In the position of minimum deviation, the angle of refraction will be
(A) 30° for both the colours (B) greater for the violet colour
(C) greater for the red colour (D) equal but not 30° for both the colours

Sol. (A)
60°
r1 r2
Minm deviation r1 = r2
& we know r1 + r2 = A
∴ 2r1 = 60°
r1 = 30°
It will be same for both colours.

29. An ideal gas is expanding such that PT2 = constant. The coefficient of volume expansion of the
gas is
1 2 3 4
(B) (C) (D)
(A)
T T T T

Sol. (C)
PT2 = C ....(i)
by gas equation PV = nRT ....(ii)
C
from equation (i) put p = 2
T
CV = nRT3

$$\frac{dV}{dT} = \frac{3nRT^2}{C}$$

$$\frac{dV}{dT} = \frac{3nRT^2}{PT^2} \quad Q\ C = PT^2$$

$$= \frac{3nRT}{PT} = \frac{3PV}{PT} \quad Q\ nRT = PV$$

$$\frac{dV}{dT} = \frac{3V}{T}$$

$$\frac{1}{V}\frac{dV}{dT} = \frac{3}{T}$$

Cofficient of volume expression $\gamma = \frac{3}{T}$

Section II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices. (A), (B), (C) and (D) out of the which ONE OR MORE is/are correct.

30. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is l . Choose the correct choice(s).

Figure :

R e gion I R e gion II R e gion III
V
l

(A) The particle enters Region III only if its velocity $V > \frac{qlB}{m}$

(B) The particle enters Region III only if its velocity $V < \frac{qlB}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = \frac{qlB}{m}$

(D) Time spent in Region II s same for any velocity V as long as the particle returns to Region I

Sol. (A), (C), (D)   Region I   Region II   Region III

× × ×
× R × ×
× × ×
V × × ×

Radius of circular path inside region II is $R = \frac{mv}{qB}$

Time spent in region II as long as the particle returns to region I is

$t = \frac{\pi R}{V} = \frac{\pi m}{qB}$

particle enters region III if

$R > l$

$\Rightarrow V > \frac{qBl}{m}$ ....(i)

Maximum path length is

$= \pi R = \pi l$

$\therefore R = l$

$\Rightarrow V = \frac{qBl}{m}$ ....(ii)

Time spent = $\frac{\pi m}{qB}$, which is independent of V.

31. In a Young s double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

(A) If $d = \lambda$, the screen will contain only one maximum

(B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

(D) If the intensity of light falling on slit 2 is increased so that if becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

Sol. (A), (B)

Δx

No matter what the relation between λ and d be, the central bright fringe will always be formed.

Besides the central maximum, the first maximum will occur when the path difference $\Delta x = \lambda$.

If $d = \lambda$, $\Delta x = d \Rightarrow$ the maximum does not fall on the screen.

Hence, option (A) is correct.

d

If $\lambda < d < 2\lambda$, then $\frac{d}{2} < \lambda < d$

$\Rightarrow \frac{d}{2} < \Delta x < d$

$\Rightarrow$ at least one more maximum will be observed on the screen.

Initially, $I_{max} = I + 4I + 2\sqrt{I.4I} = 9I$

and $I_{min} = I + 4I - 2\sqrt{I.4I} = I$

For option (c),

$I'_{max} = I + I + 2\sqrt{I.I} = 4I$

$I'_{min} = I + I - 2\sqrt{I.I} = 0$

For option (d)

$I'_{max} = 4I + 4I + 2\sqrt{4I.4I} = 16I$

$I'_{min} = 4I + 4I - 2\sqrt{4I.4I} = 0$

Option (d) is incorrect because I_{max} has increased from 9I to 16I but I_{min} has decreased from I to 0.

32. Two balls, having linear momenta $\vec{P}_1 = p\hat{i}$ and $\vec{P}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{P}'_1$ and $\vec{P}'_2$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{P}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{P}'_2 = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{P}'_1 = c_1\hat{k}$, $\vec{P}'_2 = c_2\hat{k}$

(C) $\vec{P}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{P}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{P}'_1 = a_1\hat{i} + b_1\hat{j}$, $\vec{P}'_2 = a_2\hat{i} + b_1\hat{j}$

Sol. (A), (D)

Since there is no external force on the balls, $\vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2 = 0$

’ 2

r r

(A) → p1 + ṗp= (a1 + a2) Æ+ (b1 + b2) Æ+ c1k ≠0 Q c1 ≠0

’ Æ

i j

2

r r

(B) p1 + ṗp= (c1 + c 2) k = 0 if c1 = –c 2

’ Æ

2

r r

(C) p1 + ṗp= (a1 + a2) Æ+ (b1 + b2) Æ= 0 if a1 = –a2 & b1 = –b2

’ i j

2

r r

(D) p1 + ṗp= (a1 + a2) Æ+ 2b1Æ≠ 0 Q b1 ≠ 0

’ i j

2

33. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown

inthe figure. Use this plot to choose the correct choice(s) given below.

Figure:

B /A

8

6

4

2

0 1 00 2 00 A

(A) Fusion of nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy

(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy

(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments

(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments

Sol. (B), (D)

(A) → Mass number of the resultant nucleus will be < 100. Thus, no change in B/A. Hence no energy released.

(B) → Mass number of the resultant nucleus will lie between 100 and 200. The B/A will increases ⇒ resultant nuclues is more stable ⇒ energy is released.

(C) → The daughter nuclei will have $50 < A < 100$ ⇒ B/A has decreased ⇒ stability has decreased ⇒ no release of energy.

(D) → The daughter nuclei will have $100 < A < 130$ ⇒ B/A has increased ⇒ stability has increased ⇒ energy is released.

SECTION - III
Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

34. STATEMENT-1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT-2

Resistance of a metal increases with increase in temperature.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (C)

Resistance of a metal increases with increase in temperature. So statement 2 is true. If unknown resistence increases, then the standard resistance must be increased to keep the ratio fixed (the null point remains the same).

SO STATEMENT 1 IS FALSE

35. STATEMENT-1

An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT-2

An object moving around the Earth under the influence of Earth s gravitational force is in a state of free-fall .

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (A) In the frame of reference of the space station, the gravitational force balances the centrifugal force and hence the normal reaction is zero.

Hence statement 1 is true.

As far as weightlessness is concerned above situation is equivalent to a state of free fall.

Hence 2 is true and an explanation for 1.

36. STATEMENT-1

The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain

when held vertically up, but tends to narrow down when held vertically down.
and
STATEMENT-2
In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True
Sol. (A)
This is a common observation.
Statement I is true.
$A_1 V_1 = A_2 V_2$ speed speed decreases as the fluid moves upward, due to conservation of energy.
Hence area of cross section increases.
Speed increases as the fluid moves down ward, due to conservation of energy y. Hence area of cross-section decreases.
Statement II is true and is a correct explanation of I

37. STATEMENT-1
Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.
and
STATEMENT-2
By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True
Sol. (D)
If linear acceleration along the incline is a and length of the incline is

$$l, l, l = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2l}{a}}$$

$$= \sqrt{\frac{2l}{\alpha r}}$$

$$= \frac{2l}{mg\sin\theta} \times \frac{I}{r}$$

Hence lower the moment of inertia, lower is the time taken. Hence solid cylinder will reach the bottom first.

Statement I is false

Using conservation of energy statement II is true

SECTION - IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

Paragraph for Question Nos. 38 to 40

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure.

The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic (g = 10 m/s2).

Figure :

A M

v

60° B

30° C

$3\sqrt{3}$ m

$\sqrt{3}$m

38. The speed of the block at point B immediately after it strikes the second incline is

(A) 60 m/s (B) 45 m/s (C) 30 m/s (D) 15 m/s

Sol. (B)

Let V be the speed at point B just before it strikes the second incline.

$$\therefore \frac{1}{2}MV^2 = Mg\left(\sqrt{3}\tan 60^\circ\right)$$

$$V = \sqrt{6g} \text{ m/s}$$

Let assume X axis along the incline at 30° and Y axis perpendicular to it.

$$\therefore \text{component of V along X-direction } V_x = V\cos 30^\circ = \frac{\sqrt{3}V}{2}$$

$$\text{Component of V along Y-direction } V_Y = -V\sin 30^\circ = -\frac{V}{2}$$

After the inelastic collision with second incline, Y-component will be zero and X-component will remain unchanged.

3V

∴ After collision velocity of the block, V′ = and is along X -direction

2

3 6g

3V

v̇v= = = 45 m/s

or,

2 2

39. The speed of the block at point C, immediately before it leaves the second incline is

120 m/s (B) 105 m/s (C) 90 m/s (D) 75 m/s

(A)

Sol. (B)

From conservation of energy

1 1

MVc = Mg (3 3 tan30 °) + MV ’ 2

2

2 2

⇒ Vc = 6g + V ′2

2

= 60 + 45 = 105

Vc = 105

40. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

30 m/s (B) 15 m/s (C) 0 (D) 15 m/s

(A)

Sol. (C)

If collision is perfectly elastic, x-and y-components of velocity of the block after collision, at B

3V

VB,X = VX = V = 45 m/s

2

V

VB,γ = −VY = = 15 m/s

2

vertical component (upward) of the velocity of the block,

15 co s 30 °

15

30 °

4 5 cos 30 ° + 1 5 sin 30 °

30 °

45

30 °

4 5 sin 3 0 °

= 15 cos30 ° − 45 sin30 °

=0

Paragraph for Question Nos. 41 to 43

In a mixture of H He+ gas (He+ is singly ionized He atom), H atoms and He+ ions are excited to

their respective first excited states. Subsequently, H atoms transfer their total excitation energy to

He+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.
The quantum number n of the state finally populated in He+ ions is

41.

(A) 2 (B) 3 (C) 4 (D) 5

Sol. (C)

Initially,

13.6 2 13.6
En=2,H = − .1 = − eV
2 4
2
13.6 2
=− .2 = −13.6eV
E
n=2,He+ 22

Excitation energy of H = En=2,H En=1,H

13.6 ⎛ 13.6 ⎞
=− −⎜− ⎟
⎝ 1 ⎠
4

3
= × 13.6eV
4

+
Energy of He after collisions,

⎛ ⎞
3
E + =⎜ −13.6 + × 13.6 ⎟eV
⎝ ⎠
4
He

⎛ 3⎞
= −13.6⎜ 1 − ⎟eV
⎝ 4⎠

13.6
13.6
eV = − 2 × 4
=−
4 4
=E +
n= 4,He

The wavelength of light emitted in the visible region by He+ ions after collisions with H atoms is

42.

(A) 6.5 × 10−7 m (B) 5.6 × 10−7 m (C) 4.8 × 10−7 m (D) 4.0 × 10−7 m

Sol. (C)

Due to transition from n = 4 to lower orbits, H+ will emit radiations.

e
⎡ 1 1 ⎤hc

$$\frac{hc}{\lambda} = 13.6\times Z_{He}^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\Rightarrow \lambda = \frac{hc}{13.6\times 4\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)};$$

$$= \frac{1242\times 10^{-9}}{13.6\times 4\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)} \text{ m}$$

Putting transition from $n_2 = 4$ to $n_1 = 3$

$$\lambda = \frac{1242\times 10^{-9}}{13.6\times 4\left(\frac{1}{9}-\frac{1}{16}\right)} = \frac{22.8\times 144}{7}\times 10^{-9} \approx 4.7\times 10^{-7}\text{ m}$$

So transition will lie in visible region.

For other transitions, λ is less than 300 nm.

43. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Sol. (C)

$$\frac{(K.E.)_{H,n=2}}{(K.E.)_{He^+,n=2}} = \frac{(T.E.)_{H,n=2}}{(T.E.)_{He^+,n=2}} = \frac{Z_H^2}{Z_{He^+}^2} = \frac{1}{4}$$

Paragraph for Question Nos. 44 to 46

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_l (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure :

P_0

Liq uid

H

y

44. As the bubble moves upwards, besides the buoyancy force the following forces are acting on

it

(A) Only the force of gravity
(B) The force due to gravity and the force due to the pressure of the liquid
(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
(D) The force due to gravity and the force due to viscosity of the liquid

Sol. (A)

Viscous force if present, will generate heat due to friction between bubble and liquid and hence heat exchange between the bubble & liquid contrary to what is given in the question. Hence no viscous forces are present.

45. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\dfrac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{2/5}$

(B) $T_0\left(\dfrac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{2/5}$

(C) $T_0\left(\dfrac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{3/5}$

(D) $T_0\left(\dfrac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{3/5}$

PV = nRT → ideal gas equation.

Sol. (B)

H

y

$(P_0 + PgH)V_0 = nRT_0$

$PV = nRT\ [P = P_0 + P_l g(H - y)]$

also, PV^{γ} = const.

$\Rightarrow (P_0 + P_l gH)V_0^{\gamma} = [P_0 + P_l g(H - y)]V^{\gamma}$

$\Rightarrow V = \left[\dfrac{P_0 + P_l gH}{P_0 + P_l g(H - y)}\right]^{1/\gamma} V_0$

$\Rightarrow T = \dfrac{[P_0 + P_l g(H-y)]^{1-\frac{1}{\gamma}}}{[P_0 + P_l gH]^{1-\frac{1}{\gamma}}}\, T_0;\ \left(\gamma = \dfrac{5}{3}\right)$

46. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(P0 + ρl gH)2 / 5

(A) ρlnRgT0

(P0 + ρl gy)7 / 5

ρlnRgT0

(B)

(P0 + ρl gH) [P0 + ρl g(H − y)]3 / 5

2/5

(P0 + ρl gH)3 / 5

(C) ρlnRgT0

(P0 + ρl gy)8 / 5

ρlnRgT0

(D)

(P0 + ρl gH) (P0 + ρl g(H − y)2 / 5

3/5

Sol. (B)

1/ γ

⎡ (P0 + ρgH) ⎤

FBuoyancy = ρl Vg = ρg⎢ ⎥ V0

⎣ P0 + ρg(H − y) ⎦

nRT0

Using V0 =

(P0 + ρgH)

1

−1+

ρlgnRT0 [P0 + ρgH] γ 5

FB = ; γ

[P0 + ρg(H − y)] 1/ r 3

CHEMISTRY

PART III

Section I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

47. The major product of the following reaction is

Me Br

F

PhS−Na+

——————— →

dimethylformamide

NO2

Me Br

Me SPh Me SPh Me SPh

SPh

F F SPh

(A) (B) (C) (D)

NO2

NO2 NO2 NO2

Sol. (A)

Me Br Me SPh
F F

$\xrightarrow[\text{dimethylformamide}]{PhS^-Na^+}$

NO_2 NO_2

SN2 reaction takes place inversion takes place.

48. Aqueous solution of $Na_2S_2O_3$ on reaction with Cl_2 gives

(A) $Na_2S_4O_6$ (B) $NaHSO_4$ (C) NaCl (D) NaOH

Sol. (B)

$Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$

49. Hyperconjugation involves overlap of the following orbitals

(A) $\sigma-\sigma$ (B) $\sigma-p$ (C) p p (D) $\pi-\pi$

Sol. (B)

Hyperconjunction involves overlap of σ p orbitals.

50. 2.5 mL of $\frac{2}{5}$ weak monoacidic base $\left(K_b = 1\times 10^{-12} \text{ at } 25\,°C\right)$ is titrated with $\frac{2}{15}$ M HCl in water at $25\,°C$. The concentration of H^+ at equivalences point is $\left(K_w = 1\times 10^{-14} \text{ at } 25\,°C\right)$

(A) 3.7×10^{-13} M (B) 3.2×10^{-7} M (C) 3.2×10^{-2} M (D) 2.7×10^{-2} M

Sol. (C)

2.5 ml, $\frac{2}{5}$ M BOH + $\frac{2}{15}$ M HCl, Vml

$K_b = 10^{-12}$

$M_1V_1 = M_2V_2$

$2.5 \times \frac{2}{5} = V \times \frac{2}{15}$

$V = 2.5 \times \frac{2}{5} \times \frac{15}{2} = 7.5$ ml

Number of moles of BCl formed = 1×10^{-3}

Total volume = 10×10^{-3} L

$[BCl] = \frac{1}{10} = 0.1$ M

$\left[H^+\right] = Ch = K_H C = \frac{K_w.C}{K_b}$

$10^{-14} \times 0.1$

$= 10^{-3}$

=

−12

10

= 10 × 10−4 = 3.2 × 10−2 M

51. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of

(A) nitrogen (B) oxygen (C) carbon dioxide (D) argon

Sol. (B)

Ag gets oxidised in presence of oxygen and dissolves forming the complex $Na\left[Ag(CN)_2\right]$.

52. Under the same reaction conditions, initial concentration of 1.386 mol dm 3 of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reaction is

(A) 0.5 mol−1 dm3 (B) 1.0 mol dm−3 (C) 1.5 mol dm−3 (D) 2.0 mol−1 dm3

Sol. (A)

$[A]_i = 1.38$ L mol / L $\xrightarrow[\text{in 40 sec}]{\text{first order}} \frac{1.38}{2}$ in 40 sec .

$k_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{40}$

$[A]_f = \frac{1.386}{2}$ in 20 sec. for zero order

$t_{1/2} = 20$ sec

zero order

$x = kt$

$\frac{a}{2} = kt_{1/2}$

$k_0 = \frac{a}{2t_{1/2}} = \frac{1.38}{2 \times 20}$

$\Rightarrow \frac{k_1}{k_0} = \frac{\frac{0.693}{40}}{\frac{1.38}{40}} = \frac{0.693}{1.38} = 0.502$

Section II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B) (C) and (D), out of which one or more is/are correct.

53. A solution of colourless salt H on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt(s) H is (are)

(A) NH_4NO_3 (B) NH_4NO_2 (C) NH_4Cl (D) $(NH_4)_2SO_4$

Sol. (A, B)

On addition of NaOH all ammonium salts give ammonia, but since the evolution of NH_3 resumes on addition of Zn, the salts can be NO_3^- or NO_2^- which on reduction give ammonia.

54. A gas described by van der Waals equation

(A) behaves similar to an ideal gas in the limit of large molar volumes

(B) behaves similar to an ideal gas in the limit of large pressures

(C) is characterised by van der Waals coefficients that are dependent on the identity of the gas but are independent of the temperature

(D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally.

Sol. (A, C, D)

55. The correct statement(s) about the compound given below is (are)

Cl H
CH3
H 3C
H
Cl

(A) The compound is optically active

(B) The compound possesses centre of symmetry

(C) The compound possesses plane of symmetry

(D) The compound possesses axis of symmetry

Sol. (C, D)

56. The correct statement(s) concerning the structures E, F and G is (are)

O OH CH3
H 3C H 3C H 3C
CH3 CH3
H 3C H 3C
H 3C OH

(E) (F) (G)

(A) E, F and G are resonance structure (B) E, F and G are tautomers

(C) F and G are geometrical isomers (D) F and G are diastereomers

Sol. (B, C, D)

Section III
Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

57. Statement 1:Bromobenzene upon reaction with Br_2/Fe gives 1,4-dibromobenzene as the major

product

Statement 2: In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile.

(A)Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B)Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

(C)Statement 1 is true, statement 2 is false.

(D)Statement 1 is false, statement 2 is true.

Sol. (A)

Br Br Br

Br

Br_2 , Fe

\+

(m inor product)

Br

(m ajor product)

The ortho, para - directive influence of Br is due to + mesomeric effect.

58. Statement 1:Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.

Statement 2: The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to inert pair effect .

(A)Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B)Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

(C)Statement 1 is true, statement 2 is false.

(D)Statement 1 is false, statement 2 is true.

Sol. (C)

Pb^{+4} is less stable than Pb^{2+} where Sn^{+4} is more stable than Sn^{+2}, Pb^{+4}, therefore is a stronger oxidizing agent.

The higher oxidation states of heavier elements are less stable due to inert pair effect.

59. Statement 1:For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

Statement 2: At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

(A)Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B)Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

(C)Statement 1 is true, statement 2 is false.

(D)Statement 1 is false, statement 2 is true.

Sol. (D)

For a chemical reaction at equilibrium

$\Delta G = 0, \Delta G^o \neq 0$

For a spontaneous process

$\Delta G =$ ve

60. Statement 1:The plot of atomic number (y-axis) versus number of neutrons (x-axis) for stable
nuclei shows a curvature towards x-axis from the line of $45\,°C$ slope as the atomic
number is increased.

Statement 2: Proton-proton electrostatic repulsions begin to overcome attractive forces
involving
protons and neutrons in heavier nuclides.

(A)Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement
1.

(B)Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for
statement
1.

(C)Statement 1 is true, statement 2 is false.

(D)Statement 1 is false, statement 2 is true.

Sol. (B)

Both statements are correct but statement 2 is not explanation of statement 1.

z

Stable
nucle i

$45\,°$

n

Section IV

Linked Comprehension Type

This section contains 3 paragraph. Based upon each paragraph, 3 multiple choice questions have to
be
answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Paragraph for Question Nos. 61 to 63

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when
solute
molecules are added to get homogeneous solution. These are called colligative properties.
Applications
of colligative properties are very useful in day-to-day life. One of its examples is the use of
ethylene glycol
and water mixture as anti-freezing liquid in the radiator of automobiles

A solution M is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture
is 0.9

Given :

Freezing point depression constant of water $K_f(\text{water}) = 1.86\ \text{K kg mol}^{-1}$

Freezing point depression constant of ethanol $K_f(\text{ethanol}) = 2.0\ \text{K kg mol}^{-1}$

Boiling point elevation constant of water $K_b(\text{water}) = 0.52\ \text{K kg mol}^{-1}$

( )

ethanol
= 1.2 K kg mol^{-1}

Boiling point elevation constant of ethanol K_b
Standard freezing point of water = 273 K
Standard freezing point of ethanol = 155.7 K
Standard boiling point of water = 373 K
Standard boiling point of ethanol = 351.5 K
Vapour pressure of pure water = 32.8 mm Hg
Vapour pressure of pure ethanol = 40 mm Hg
Molecular weight of water = 18 g mol^{-1}
Molecular weight of ethanol = 46 g mol^{-1}

In answering the following questions, consider the solutions to be ideal dilute solutions and solutes to be non-volatile and non-dissociative.

61. The freezing point of the solution M is
(A) 268.7 K (B) 268.5 K (C) 234.2 K (D) 150.9 K

Sol. (D)
Let total moles be 1, then
$n_{ethanol} = 0.9$
mass of ethanol = 41.4 g = 41.4×10^{-3} kg
$n_{water} = 0.1$

molality, m = $\frac{0.1}{41.4} \times 10^3 = 2.4$ mol/kg.
$\Delta T_f = K_f m = 2 \times 2.4 = 4.8$
$T_{f\,ethanol} = 155.7 - 4.8 = 150.9$ K

62. The vapour pressure of the solution M is
(A) 39.3 mm Hg (B) 36.0 mm Hg (C) 29.5 mm Hg (D) 28.8 mm Hg

Sol. (B)
$\frac{40 - P_s}{40} = 0.1$
P_S = 36 mm of Hg.

63. Water is added to the solution M such that the mole fraction of water in the solution becomes 0.9.
The boiling point of this solution is
(A) 380.4 K (B) 376.2 K (C) 375.5 K (D) 354.7 K

Sol. (B)
Let no of moles be 1
$n_{H_2O} = 0.9$, $n_{ethanol} = 0.1$
mass of H_2O = 0.9 ×18 g = 16.2 g = 0.0162 kg
molality = $\frac{0.1}{0.0162}$ mol / kg = 06.17 mol/kg
$\Delta T_b = k_b.m$
= 0.52 × 6.17 = 3.2
Boiling point of solution = 373 + 3.2 = 376.2 K

Paragraph for Question Nos. 64 to 66

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water.

Nitrates are difficult to reduce under the laboratory conditions but microbes do it easily. Ammonia forms
large number of complexes with transition metal ions. Hybridization easily explains the ease of sigma
donation capability of NH_3 and PH_3. Phosphine is a flammable gas and is prepared from white phosphorous.

64. Among the following, the correct statement is

(A) Phosphates have no biological significance in humans
(B) Between nitrates and phosphates, Phosphates are less abundant in earth s crust
(C) Between nitrates and phosphates, nitrates are less abundant in earth s crust
(D) Oxidation of nitrates is possible in soil

Sol. (C)

65. Among the following, the correct statement is

(A) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies spherical s orbital and is less directional
(B) Between NH_3 and PH_3, PH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(C) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(D) Between NH_3 and PH_3, PH_3 is a better electron donor because the lone pair of electrons occupies spherical s orbital and is less directional

Sol. (C)

White phosphorus on reaction with NaOH gives PH_3 as one of the products. This is a

66.

(A) dimerization reaction (B) disproportionation reaction
(C) condensation reaction (D) precipitation reaction

Sol. (B)

+1
0 3

$P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

Disproportionation

Paragraph for Question Nos. 67 to 69

In the following reaction sequence, products I, J and L are formed. K represents a reagent.

1. M g/ether
1. N a BH 4 2: C O 2 K
H ex-3-ynal I J Me H2
L
+
2. P B r 4 Cl
3. H 3 O P d/Ba SO 4
quinoline
O

67. The structure of the product I is

(A) M e Br
(B) M e

Br

(C) M e Br

(D) M e Br

Sol. (D)

68. The structures of compounds J and K, respectively, are

(A) M e C O O H and S O C l2

O H a n d S O 2C l2

(B) M e

O

(C) M e and SOCl2

COOH

(D) M e C O O H and CH3SO2Sl

Sol. (A)

69. The structure of product L is

(A) M e CHO

(B) M e CHO

CHO

(C) M e

(D) M e CHO

Sol. (C)

Explanation for questions 67 to 69:

CH_3 CH_2 C≡ C CH_2 CHO

↓$NaBH_4$

CH CH_2 C≡ C CH_2 CH_2OH

↓PBr_3

CH_3 CH_2 C≡ C CH_2 CH_2 Br (I)

↓Mg / ether

CH_3 CH_2 C≡ C CH_2 CH_2 Mg.Br

↓CO_2

CH_2 CH_2 C≡ C $CH_2CH_2COOMgBr$

↓H_3O^+

CH_3 CH_2 C≡ C CH_2 CH_2COOH (J)

↓$SOCl_2$ (K)

CH_3 CH_2 C≡ C CH_2 CH_2COCl

↓H_2 ,Pd / $BaSO_4$ quinoline

H H

| |

CH_3 CH_2 C = C CH_2 CH_2 CHO (L)