

PART I
SECTION I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. If $0 < x < 1$, then

$$\frac{1 + x^2}{x} \left[\left\{ x \cos(\cot^{-1} x) + \sin(\cot^{-1} x) \right\}^2 - 1 \right]^2 =$$

- (A) $\frac{1+x^2}{1+x^2}$ (B) x (C) $x \sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Sol. (C)

The given expression

$$\frac{1+x^2}{x} \left[\left\{ x \cos(\cot^{-1} x) + \sin(\cot^{-1} x) \right\}^2 - 1 \right]^2$$

$$= \frac{1+x^2}{x} \left[\left\{ \frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}} \right\}^2 - 1 \right]^2$$

$$= \frac{1+x^2}{x} \left[\frac{x^2 + 1 + 2x}{1+x^2} - 1 \right]^2$$

$$= \frac{1+x^2}{x} \left[\frac{x^2 + 1 + 2x - 1 - x^2}{1+x^2} \right]^2$$

$$= \frac{1+x^2}{x} \left[\frac{2x}{1+x^2} \right]^2$$

$$= \frac{1+x^2}{x} \cdot \frac{4x^2}{(1+x^2)^2}$$

$$= \frac{4x}{1+x^2}$$

~~AAA~~

2. The edges of a parallelepiped are of unit length and are parallel to non-coplanar unit vectors a, b, c

such that $a \cdot b = b \cdot c = c \cdot a =$

$$\frac{1}{2}$$

Then, the volume of the parallelepiped is

- (A) $\frac{1}{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{2\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

Sol. (A)

$$Q_a = \begin{vmatrix} 1 & 1 & 2 \\ i & j & k \\ 2 & 2 & 3 \end{vmatrix}$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 1

$$\text{Volume} = \frac{1}{6} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 2 & 2 & 1 \end{vmatrix} = \frac{1}{6} \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 2 & 2 & 3 \end{vmatrix}$$

3. Consider the two curves

$$C_1 : y^2 = 4x$$

$$C_2 : x^2 + y^2 - 6x + 1 = 0$$

Then,

C1 and C2 touch each other only at one point

(A)

C1 and C2 touch each other exactly at two points

(B)

C1 and C2 intersect (but do not touch) at exactly two points

(C)

C1 and C2 neither intersect nor touch each other

(D)

Sol. (B)

$$\therefore y = 4x \dots\dots(i) \text{ and } x^2 + y^2 - 6x + 1 = 0 \dots\dots (ii)$$

Solving (i) and (ii)

$$x^2 + 4x - 6x + 1 = 0 \Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x - 1)^2 = 0$$

$$\Rightarrow x = 1$$

$$\therefore y = \pm 2.$$

\(\therefore\) C1 and C2 touch each other exactly at two points.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 2

$$\left\{ \begin{array}{l} (2+x)^3, \quad -3 < x \leq -1 \\ \end{array} \right.$$

The total number of local maxima and local minima of the function $f(x) = \left\{ \begin{array}{l} (2+x)^3, \quad -3 < x \leq -1 \\ x^2/3, \quad -1 < x \leq 2 \end{array} \right.$

4.

$$\left\{ \begin{array}{l} x^2/3, \quad -1 < x \leq 2 \end{array} \right.$$

is

(A) 0

(B) 1

(C) 2

(D) 3

Sol. (C)

The graph of the function is

$$\begin{matrix} & & 0 & 2 \\ & & | & | \\ 2 & & 1 & \end{matrix}$$

There is one local maxima and one local minima.

Let a and b non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

5.

represents

- (A) four straight lines, when $c = 0$ and a, b are of the same sign
- (B) two straight lines and a circle, when $a = b$, and c is of sign opposite to that of a
- (C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
- (D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to

that

of a

Sol. (B) $x^2 - 5xy + 6y^2 = 0$

$$\Rightarrow x^2 - 3xy - 2xy + 6y^2 = 0$$

$$\Rightarrow x(x - 3y) - 2y(x - 3y) = 0$$

$$\Rightarrow (x - 3y)(x - 2y) = 0 \Rightarrow \text{two straight lines.}$$

and when $a = b$ and sign of c is opposite of a the equation $ax^2 + by^2 + c = 0$ represent circle.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 3

$$(x - 1)^n$$

Let $g(x) = \frac{1}{(x - 1)^n}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand

6.

$$\log \cos^m(x - 1)$$

derivative of $x - 1$ at $x = 1$.

If $\lim_{x \rightarrow 1+} g(x) = p$, then

$$x \rightarrow 1+$$

(A) $n = 1, m = 1$

(B) $n = 1, m = -1$

(C) $n = 2, m = 2$

(C) $n > 2, m = n$

Sol. (C)

Q $p = -1$,

$$\frac{1}{(x - 1)^n}$$

Lt

$$+ g(x) = -1 \Rightarrow \lim_{x \rightarrow 1} \frac{1}{(x - 1)^n} = -1$$

$$x \rightarrow 1$$

$$\lim_{x \rightarrow 1} \frac{1}{(x - 1)^n} + \log \cos^m(x - 1)$$

$$\frac{n - 1}{n(x - 1)}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{1}{(x - 1)^n} = -1$$

$$+ \lim_{x \rightarrow 1} \frac{1}{(x - 1)^{m - 1}}$$

$$x \rightarrow 1 \frac{1}{(x - 1)^{m - 1}} \times m \cos^{m - 1}(x - 1) \times (-\sin(x - 1))$$

$$\frac{m}{\cos(x - 1)}$$

$$\frac{n - 2}{n(x - 1)} \cdot (x - 1)$$

Lt

$$\frac{n - 2}{n(x - 1)}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{1}{(x - 1)^n} = -1.$$

+

$$\Rightarrow \lim_{x \rightarrow 1} \frac{1}{(x - 1)^n} = -1$$

$$\lim_{x \rightarrow 1} \frac{1 - \sin(x-1)}{\cos(x-1)} \times \lim_{x \rightarrow 1} \frac{x^m}{1+x^m}$$

Limit to be exist $n - 2 = 0 \Rightarrow n = 2$. and $m = 2$

SECTION - II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

7. Let $S_n = \sum_{k=0}^{n-1} (n^2 + kn + k^2)$ and $T_n = \sum_{k=1}^n (n^2 + kn + k^2)$. Then,

- (A) $S_n < T_n$
- (B) $S_n > T_n$
- (C) $T_n < S_n$
- (D) $T_n > S_n$

Sol.

$$S_n = \sum_{k=0}^{n-1} (n^2 + kn + k^2)$$

$$= \sum_{k=1}^n (n^2 + (k-1)n + (k-1)^2)$$

$$= \sum_{k=1}^n (n^2 + kn - n + k^2 - 2k + 1)$$

$$= \sum_{k=1}^n (n^2 + kn + k^2) - \sum_{k=1}^n (n - 2k + 1)$$

$$= S_n - \sum_{k=1}^n (n - 2k + 1)$$

$$= S_n - (n^2 - 2 \sum_{k=1}^n k + n)$$

$$= S_n - (n^2 - 2 \cdot \frac{n(n+1)}{2} + n)$$

$$= S_n - (n^2 - n^2 - n + n)$$

$$= S_n - 0$$

$$S_\infty = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} S_n$$

$$\text{At } x = \frac{1}{2}, f' \Big|_{\left(\frac{1}{2}\right)} = f' \Big|_{\left(\frac{1}{2}\right)} \Rightarrow f' \Big|_{\left(\frac{1}{2}\right)} = 0$$

$$\text{At } x = \frac{1}{4}, f' \Big|_{\left(\frac{1}{4}\right)} = f' \Big|_{\left(\frac{1}{4}\right)} \Rightarrow f' \Big|_{\left(\frac{1}{4}\right)} = 0$$

$$\therefore f' \Big|_{\left(\frac{1}{4}\right)} = f' \Big|_{\left(\frac{1}{2}\right)} = f' \Big|_{\left(\frac{3}{4}\right)} = 0$$

\therefore By Rolle's theorem, there exists.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 6

c_1, c_2 between $\left(\frac{1}{4}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{3}{4}\right)$ respectively.

Such that $f''(c_1) = f''(c_2) = 0$

$$\int_{\frac{1}{2}}^{\frac{1}{2}} f \Big|_{\left(x + \frac{1}{2}\right)} \sin x \, dx$$

$$\int_{\frac{1}{2}}^{\frac{1}{2}} f \Big|_{\left(x + \frac{1}{2}\right)} \sin(x) \, dx$$

$$= \int_{\frac{1}{2}}^{\frac{1}{2}} \left(\frac{1}{2}\right)$$

$$= \int_{\frac{1}{2}}^{\frac{1}{2}} f \Big|_{\left(x + \frac{1}{2}\right)} \sin x \, dx$$

$$\left(\frac{1}{2}\right)$$

$$\text{as } f(x) = f\left(\frac{1}{2} - x\right) \text{ let } x = \frac{1}{2} - x \Rightarrow f \Big|_{\left(x + \frac{1}{2}\right)} = f \Big|_{\left(\frac{1}{2} - x\right)}$$

$$\int_{\frac{1}{2}}^{\frac{1}{2}} \left(\frac{1}{2}\right)$$

$$\Rightarrow \int_{\frac{1}{2}}^{\frac{1}{2}} f \Big|_{\left(x + \frac{1}{2}\right)} \sin x \, dx = 0$$

$$\int_{\frac{1}{2}}^{\frac{1}{2}} f(t) e^t \, dt$$

$$\int_0^{1/2} \sin \pi t \, dt = \int_0^{1/2} \sin \pi(1-x) e^{\sin \pi x} dx$$

$$= \int_{1/2}^1 \sin \pi x e^{\sin \pi x} dx$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 7

9. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the

circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

- (A) $PS + ST < \frac{1}{2} QS \times SR$
- (B) $PS + ST > \frac{1}{2} QS \times SR$
- (C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$
- (D) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

Sol. (B,D)

$$Q PS \times ST = QS \times SR$$

Apply GM > HM for PS and ST

$$PS \times ST > \frac{1}{2} \left(\frac{1}{PS} + \frac{1}{ST} \right) QS \times SR$$

and apply AM > GM on QS, SR

$$\frac{QR + SR}{2} > QS \times SR$$

$$\frac{QR}{2} > QS \times SR$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$$

PS ST

Let P(x1, y1) and Q (x2, y2), y1 < 0, y2 < 0, be the end points of the latus rectum of the ellipse x² +

10.

4y² = 4. The equations of parabolas with latus rectum PQ are

- (A) x² + 2√3 y = 3 + 3
- (B) x² - 2√3 y = 3 + 3
- (C) x² + 2√3 y = 3 - 3
- (D) x² - 2√3 y = 3 - 3

Sol. (B, C)

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

(0,1) (√3,0) (√3,0)

(-2,0)

(2,0) P (√3, -1/2)

Q (√3, 1/2) (0, -1)

Focus (0, 1)

Directrix y = -1 ± 3

∴ Equation of parabola becomes

$$(x - 0)^2 + \left(\frac{y + 1}{2}\right)^2 = \left(\frac{y + 1}{2} \pm 3\right)^2$$

$$= \left(\frac{y + 1}{2}\right)^2 + 3 \pm 2 \cdot 3 \left(\frac{y + 1}{2}\right)$$

$$x = 3 \pm 3(2y + 1)$$

$$x - 2\sqrt{3}y = 3 \pm 3$$

ie. x² - 2√3y = 3 + 3

or x² + 2√3y = 3 - 3

SECTION - III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out

of which ONLY ONE is correct.

Let f and g be real valued functions defined on interval (-1, 1) such that 'gg'(x) is continuous,

11.

$g(0) \neq 0, \dot{g}(0) = 0, \dot{g}'(0) \neq 0,$ and $f(x) = g(x) \sin x.$

STATEMENT - 1 : $\lim_{x \rightarrow 0} [g(x)\cot x - g(0)\csc x] = f''(0).$

and

STATEMENT - 2 : $f'(0) = g(0).$

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation

for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct

explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (B)

$$f(x) = g(x) \sin x$$

$$f'(x) = g(x)\cos x + \dot{g}(x)\sin x$$

$$f'(0) = g(0) \dots\dots\dots(\text{I})$$

$$f''(x) = -g(x)\sin x + \dot{g}(x)\cos x + \dot{g}'(x)\cos x + g\dot{g}'(x)\sin x$$

$$= -g(x)\sin x + 2\dot{g}(x)\cos x + \dot{g}'(x)\sin x$$

$$f''(0) = 0$$

$$g(x)\cos x - g(0)$$

lim

$$\sin x$$

$$x \rightarrow 0$$

$$0$$

form

$$0$$

$$\dot{g}(x)\cos x - g(x)\sin x$$

$$= 0 = f''(0)$$

lim

$$\cos x$$

$$x \rightarrow 0$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 10

12. Consider three planes

$$P1 : x - y + z = 1$$

$$P2 : x + y - z = 1$$

$$P3 : x - 3y + 3z = 2.$$

Let L1, L2, L3 be the lines of intersection of the planes P2 and P3, P3 and P1, and P1 and P2, respectively.

STATEMENT - 1 : At least two of the lines L1, L2 and L3 are non-parallel.

and

STATEMENT - 2: The three planes do not have a common point.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation

for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation

for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (D)

Gives L1, L2, L3 have direction ratios 0 : 1 : 1

all are parallel

Here D = 0 = D1

D2 = D3 = 2

No solution

13. Consider the system of equations

$$ax + by = 0, cx + dy = 0, \text{ where } a, b, c, d \in \{0, 1\}.$$

3

STATEMENT - 1 : The probability that the system of equation has a unique solution is

8

and

STATEMENT - 2 : The probability that the system of equations has a solution is 1.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation

for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(C) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (B)

There are 16 determinants of entry 0 and 1.

$$\text{and } \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \text{ are six non-zero determinants.}$$

6 3

= .

∴ Probability that system has unique solution is

16 8

⇒ Statement 1 is correct.

and Q It is a homogeneous equation

⇒ System has a solution ⇒ Probability is 1.

14. Consider the system of equations

$$x - 2y + 3z = 1$$

$$x + y - 2z = k$$

$$x - 3y + 4z = 1.$$

STATEMENT - 1 : The system of equation has no solution for $k \neq 3$.

and

$$\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix}$$

$$-1 - 2k \neq 0, \text{ for } k \neq 3.$$

STATEMENT - 2 : The determinant

$$\begin{vmatrix} 1 & 4 & 1 \end{vmatrix}$$

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation

for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (A)

$$\begin{aligned}
 & \begin{array}{ccc} 1 & 2 & 3 \\ QD = & 1 & 1 & 2 = 0 \\ & 1 & 3 & 4 \\ & -1 & 2 & 3 \end{array} \\
 D1 = k & \begin{array}{ccc} 1 & 2 \\ & 1 & 3 & 4 \end{array} \\
 = & (4 - 6) + 2(4k + 2) + 3(-3k - 1) \\
 = & 2 + 8k + 4 - 9k - 3 \\
 = & k + 3 \neq 0, k \neq -3. \\
 \Rightarrow & \text{System has no solution for } K \neq -3 \\
 \Rightarrow & \text{Statement 1 is correct.}
 \end{aligned}$$

Again

$$\begin{aligned}
 & \begin{array}{ccc} 1 & 3 & 1 \\ D2 = & 1 & 2 & K \\ & 1 & 4 & 1 \end{array} \\
 \Rightarrow D2 = & 1(2 - 4K) - 3(1 - K) - 1(4 + 2) \\
 = & 2 - 4K + 3 + 3K + 4 - 2 \\
 = & K + 3 \neq 0, k \neq -3 \\
 \Rightarrow & \text{Statement 2 is true and correct explanation of statement 1}
 \end{aligned}$$

Section IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be

answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Paragraph for question no. 15 to 17.

Let A, B, C be three sets of complex numbers as defined below.

$$A = \{z : \text{Im } z \geq 1\}$$

$$B = \{z : z - 2 - i = 3\}$$

$$\{ \quad \quad \quad \}$$

$$C = \{z : \text{Re}(1 - i)z = 2\}$$

The number of elements in the set $A \cap B \cap C$ is

15.

(D) ∞

(A) 0

(B) 1

(C) 2

Sol. (B)

To answer this question, we need to draw the area/region as marked by A, B & C

$$\text{For } A = \{z : \text{Im}(z) \geq 1\}$$

A denotes area/region (in Argand plane) beyond $y \geq 1$

$$\text{For } B = \{z : z - 2 - i = 3\}$$

B denotes point on the circle (in Argand plane) with centre at (2, 1) and radius of 3 units.

$$\{ \quad \quad \quad \}$$

$$\text{For } C = \{z : \text{Re}(1 - i)z = 2\}$$

C denotes points on the line $x + y = 2$

P

(0,1)

(2,1)

y=1
x

Page 14

Hence, $A \cap B \cap C$ will be only one point P which is intersection of line $x + y = 2$ and circle :
 $(x - 2)^2 + (y - 1)^2 = 9$.

Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

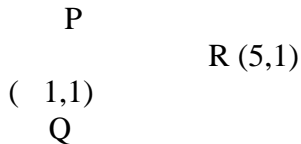
16.

- (A) 25 and 29 (B) 30 and 34 (C) 35 and 39 (D) 40 and 44

Sol. (C)

z is the point P as per solution of question 15.

Now, if we look carefully at $|z + 1 - i|^2 + |z - 5 - i|^2$, we can see that it is nothing but sum of square of distance between P and Q(-1, 1) and P and R(5, 1).



As is evident that Q and R are on circle itself at two ends of diameters.

Hence,

$$PQ^2 + PR^2 = QR^2$$

$$\text{As } QR = 6$$

$$\therefore QR^2 = 36 = PQ^2 + PR^2$$

Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|\omega - 2 - i| < 3$. Then, $|z - \omega +$

3

17.

lies between

- (A) 6 and 3 (B) 3 and 6 (C) 6 and 6 (D) 3 and 9

Sol. (B)

Now ω is defined by $|\omega - 2 - i| < 3$,

Which means ω is all the points inside circle represented by $(x - 2)^2 + (y - 1)^2 = 9$ in the

Argand

$$2 \quad 2$$

plane.

Now, we have to find out coordinates of point P. to find the range of $|z - \omega + 3|$

To find coordinates of point P we have to solve the following equations simultaneously.

$$x + y = 2$$

$$(x - 2)^2 + (y - 1)^2 = 9$$

on solving, we get

$$2x^2 - 2x - 1 + 2x - 2x - 1 + 2 = 0$$

$$x^2 - 2x - 1 + 2x - 2x - 1 + 2 = 0$$

$$x = \frac{(1 + 2) \pm \sqrt{7 + 6}}{2} \quad (+ve, \text{ sign to be ignored as P is in second quadrant})$$

$$x = \frac{2.4 - 15.4}{2} = -0.76$$

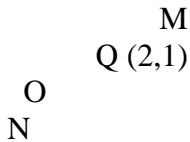
$$\text{as } x + y = 2 \Rightarrow y = 2.16$$

$$P \equiv (-0.76, 2.16)$$

$$z = \sqrt{(0.76)^2 + (2.16)^2} = \sqrt{0.58 + 4.67} = \sqrt{5.25} = 2.3$$

$$z + 3 = 5.3$$

Range of ω can be found by finding OM and ON



$$OM = \text{distance from } O(0, 0) \text{ to centre } (2, 1) + \text{Radius of circle denoted by } \sqrt{(x - 2)^2 + (y - 1)^2} = 9$$

$$\therefore OM = 4 + 1 + 3 = 5 + 3$$

$$\therefore OM = 5.3$$

$$\text{Further } ON = MN \quad OM = 2 \times 3 = 3$$

$$\therefore ON = 0.7$$

$$\text{Hence } 0.7 < \omega < 5.3$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 16

Paragraph for question no. 18 to 20

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.

If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

18. If $f''(2) = 2$, then $f''(-10) =$

(A) $\frac{3}{2}$ (B) $-\frac{4}{3}$ (C) $\frac{4}{3}$ (D) $-\frac{7}{3}$

Sol. (A)

$$y^3 - 3y + x = 0$$

$$\therefore 3y^2 y' - 3y' + 1 = 0, \quad y' = \frac{1}{3(1 - y^2)}$$

$$\Rightarrow 3y'' y^2 - 3y'' + 6y(y') = 0$$

$$\Rightarrow y'' y^2 - 1 = -2y(y')$$

$$\Rightarrow y'' = \frac{2y}{(1 - y)^2} - \frac{1}{3(1 - y^2)^2} = \frac{2y}{9(1 - y^2)^2} \quad ()$$

$$()$$

$$Q \int_{-10}^2 2x^2 dx = 2 \left[\frac{2x^3}{3} \right]_{-10}^2 = \frac{4}{3} (2^3 - (-10)^3) = \frac{4}{3} (8 + 1000) = \frac{4}{3} \times 1008 = 1344$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 17

19. The area of the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, where

$$-\infty < a < b < -2 \text{ is}$$

- (A) $\int_a^b x f(x) dx + b f(b) - af(a)$
- (B) $\int_a^b x f(x) dx + b f(b) - af(a)$
- (C) $\int_a^b x f(x) dx - b f(b) + af(a)$
- (D) $\int_a^b x f(x) dx - b f(b) + af(a)$

Sol. (A)

$$Q \int x f(x) dx = x f(x) - \int x f'(x) dx$$

$$\therefore \text{Required area} = \int_a^b f(x) dx$$

$$= \left[x f(x) \right]_a^b - \int_a^b x f'(x) dx$$

$$= \left[x f(x) \right]_a^b - \int_a^b x f'(x) dx$$

$$= \left[x f(x) \right]_a^b - \int_a^b x f'(x) dx$$

b

$$\int_a^x \frac{dx}{(f(x))^2 - 1} + b f(b) - a f(a)$$

$$\int_{-1}^1 g'(x) dx =$$

20. (A) $2g(-1)$ (B) 0 (C) $2g(1)$ (D) $2g(1)$

Sol. (D)

$$\int_{-1}^1 g'(x) dx = [g(x)]_{-1}^1 - 1$$

$$= g(1) - g(-1)$$

$$= g(1) - (-g(1))$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 18

$$= 2g(1)$$

Since given curve $y^3 - 3y + x = 0$ is symmetric about origin

\therefore if $y = g(x)$

$$\Rightarrow y = g(-x)$$

$$\Rightarrow g(-1) = -g(1)$$

Paragraph for question no. 21 to 23

21. A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the

$$3x + y - 6 \text{ and}$$

sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation

the point D is $(\frac{3}{2}, \frac{3}{2})$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

$$(x - \frac{3}{2})^2 + (y - \frac{3}{2})^2 = 1$$

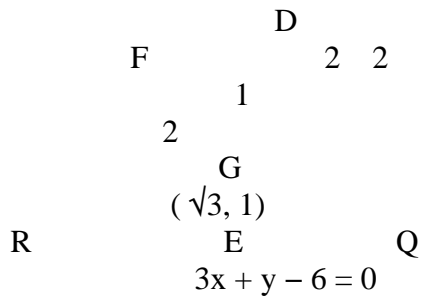
(A) $x - 2\sqrt{3} + (y - 1) = 1$ (B) $x - 2\sqrt{3} + |y + 1| = 1$

(C) $x - 3 + (y + 1) = 1$ (D) $x - 3 + (y - 1) = 1$

Sol. (D)

P

$$3\sqrt{3}, 3$$



Equation of PQ

Radius of circle = 1

a

$r = \frac{a}{2\sqrt{3}}$, a = length of side of equilateral triangle.

IIT-JEE 2008 SOLUTIONS PAPER 1

$\Rightarrow a = 2\sqrt{3}$

Equation of DR is $x - 3y + k = 0$

$\Rightarrow \frac{3^2 + 3^2}{2} + k = 0 \Rightarrow k = 0$

\therefore Equation of DR is $x - 3y = 0$

\therefore Co-ordinates of G $\equiv (3, 1)$ as origin and centre of circle lie in the same side.

$\frac{(x-3)^2}{2} + (y-1)^2 = 1$

\therefore Equation of circle $x^2 - 6x + 3y^2 - 2y - 2 = 0$

22. Points E and F are given by

(A) $\left(\begin{matrix} 3 & 3 \\ 2 & 2 \\ 3 & 0 \end{matrix} \right)$

(B) $\left(\begin{matrix} 3 & 1 \\ 2 & 2 \\ 3 & 0 \end{matrix} \right)$

(C) $\left(\begin{matrix} 3 & 3 \\ 2 & 2 \\ 3 & 0 \end{matrix} \right) \left(\begin{matrix} 3 & 1 \\ 2 & 2 \\ 3 & 0 \end{matrix} \right)$

(D) $\left(\begin{matrix} 3 & 3 \\ 2 & 2 \\ 3 & 0 \end{matrix} \right) \left(\begin{matrix} 3 & 1 \\ 2 & 2 \\ 3 & 0 \end{matrix} \right)$

Sol. (A)

Slope of PQ is $-\frac{3}{1}$

$\therefore \cos \theta = -\frac{1}{2}, \sin \theta = \frac{3}{2}$

$\therefore x = \frac{3 \pm 3\sqrt{1 - \frac{1}{4}}}{2} = \frac{3 \pm 3\sqrt{\frac{3}{4}}}{2} = \frac{3 \pm \frac{3\sqrt{3}}{2}}{2}$

$= \frac{3}{2} \pm \frac{3\sqrt{3}}{4}$

$y = \frac{3 \pm 3\sqrt{1 - \frac{1}{4}}}{2} = \frac{3 \pm \frac{3\sqrt{3}}{2}}{2}$

Page 20

$$\frac{3 \times 3}{2 \times 2} = \frac{\pm}{2} = 3, 0$$

\therefore Co-ordinates of P and Q are $(3, 3)$ and $(2, 3, 0)$

Co-ordinates of R is (h, k)

$$\frac{2 \times 3 \times 3}{1 \times h + 2} = 3$$

$$h + 3 \times 3 = 3 \times 3 \Rightarrow h = 0$$

$$\frac{1 + k \times 2 \times 2}{2} = 1$$

and

$$\frac{k + 3 = 3}{1 + 2} \Rightarrow k = 0$$

$\therefore R = (0, 0)$

Coordinates of E and F are

$$\left(\frac{3 + 0}{2}, \frac{3 + 0}{2} \right) \text{ and } \left(\frac{2 \times 3 + 0}{2}, \frac{0 + 0}{2} \right)$$

$$\left(\frac{3}{2}, \frac{3}{2} \right) \text{ and } (3, 0)$$

i.e. $\left(\frac{3}{2}, \frac{3}{2} \right), (3, 0)$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 21

23. Equations of the sides QR, RP are

$$(A) y = \frac{2}{3}x + 1, y = -\frac{2}{3}x - 1 \quad (B) y = -\frac{1}{3}x, y = 0$$

$$(C) y = \frac{3}{2}x + 1, y = -\frac{3}{2}x - 1 \quad (D) y = 3x, y = 0$$

Sol. (D)

Equation of QR and PR are

$$y = 3x \text{ and } y = 0$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 22

PHYSICS

PART II

SECTION I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D),

Student			Time period (s)	
	pendulum (cm)	oscillations (n)	oscillations (s)	
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I , E_{II} and E_{III} are the percentage errors in g , i.e., $\left(\frac{\Delta g}{g} \right) \times 100$ for students, I, II and III, respectively,

respectively,

(A) $E_I = 0$

(B) E_I is minimum

(C) $E_I = E_{II}$

(D) E_{II} is maximum

Sol. (B)

We know

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2 \frac{l}{g}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$\frac{\Delta g}{g} \times 100 = \left[\frac{\Delta l}{l} + 2 \frac{\Delta T}{T} \right] \times 100$$

Exp I

$$\left[\frac{.1}{64} + 2 \times \frac{.1}{16} \right] \times 100$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 24

$$= \left[\frac{.1}{64} + 2 \times \frac{.1}{16} \right] \times 100 = \left[\frac{.1}{64} + \frac{.2}{16} \right] \times 100 = \left[\frac{.1}{64} + \frac{.8}{64} \right] \times 100 = \frac{.9}{64} \times 100 = 1.40625 \times 100 = 140.625 \approx 141\%$$

because No. of oscillation taken by student I is 8 as compare to 4 of student II.

\therefore Student I is more accurate as compare to II

So $E_I \neq E_{II}$

Exp II:

$$\frac{\Delta g}{g} \times 100 = \left[\frac{\Delta l}{l} + 2 \frac{\Delta T}{T} \right] \times 100$$

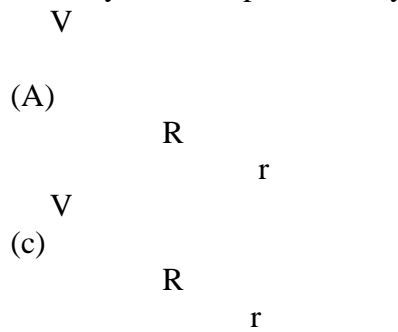
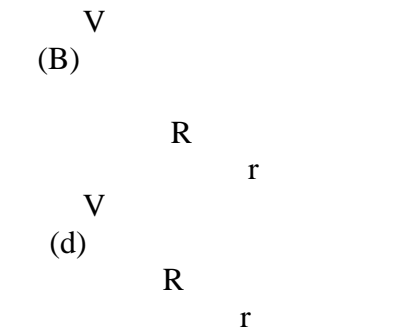
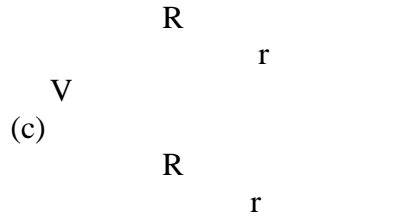
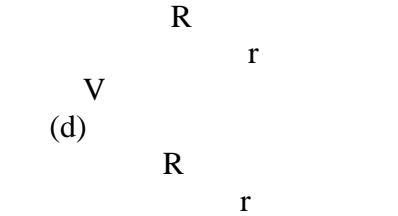
$$= \left[\frac{.9}{20} + 2 \times \frac{.1}{9} \right] \times 100 = \left[\frac{.9}{20} + \frac{.2}{9} \right] \times 100 = \left[\frac{81}{180} + \frac{40}{180} \right] \times 100 = \frac{121}{180} \times 100 = 67.22\%$$

27. A spherically symmetric gravitational system of particles has a mass density

$$\rho = \begin{cases} \rho & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

ρ_0

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

- (A) 
- (B) 
- (c) 
- (d) 

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 25

Sol. (C)

$$\rho = \begin{cases} \rho_0 & r \leq R \\ \rho_0 & r > R \end{cases}$$

Gravitational field inside the sphere of distance r from the centre

$$g = \frac{GM}{R^3} r$$

where $M = \frac{4}{3} \pi R^3 \rho_0$

$$g = \frac{G \pi R^3 \rho_0}{3} r$$

$$g = \frac{4}{3} \pi G \rho_0 r$$

\therefore Inside the sphere $mg = \frac{mv^2}{r}$

$$\Rightarrow g = \frac{v^2}{r}$$

$$\frac{4}{3} \pi G \rho_0 r^2 = v^2$$

$$\Rightarrow v = \frac{2}{3} \sqrt{\pi G \rho_0} r$$

$\Rightarrow v \propto r$
& for $r > R$

$$g = \frac{GM}{r^2} = \frac{G \times \frac{4}{3} \pi R^3 \rho_0}{r^2}$$

so outside the sphere

$$mg = \frac{mv^2}{r} \quad g = \frac{v^2}{r}$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 26

$$\frac{4}{3} \pi R^3 \rho_0 v^2 = \frac{r^2}{r}$$

$$\Rightarrow v \propto r$$

so graph will be like this

R r

28. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be
- (A) 30° for both the colours (B) greater for the violet colour
 (C) greater for the red colour (D) equal but not 30° for both the colours

Sol. (A)

$$\frac{60^\circ}{r_1 + r_2}$$

Minm deviation $r_1 = r_2$
 & we know $r_1 + r_2 = A$
 $\therefore 2r_1 = 60^\circ$
 $r_1 = 30^\circ$

It will be same for both colours.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 27

- An ideal gas is expanding such that $PT^2 = \text{constant}$. The coefficient of volume expansion of the
- 29.

gas is

1	2	3	4
(A)	(B)	(C)	(D)
T	T	T	T

Sol. (C)

$PT^2 = C$ (i)
 by gas equation $PV = nRT$ (ii)

$$P = \frac{C}{T^2}$$

from equation (i) put $p = \frac{C}{T^2}$

$$CV = nRT^3$$

$$\begin{aligned}
 dV &= 3nRT^2 \\
 C & \\
 dT & \\
 dV &= 3nRT^2 \\
 Q C &= PT^2 \\
 &= \\
 &PT^2 \\
 dT & \\
 3nRT &= 3PV \\
 &= \\
 PT & \\
 dV &= 3V \\
 &= \\
 dT &= T \\
 1 &= dV^3 \\
 &= \\
 V &= dT^T
 \end{aligned}$$

Coefficient of volume expansion $\gamma = \frac{3}{T}$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 28

Section II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices. (A),

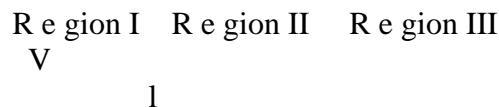
(B), (C) and (D) out of the which ONE OR MORE is/are correct.

30. A particle of mass m and charge q , moving with velocity V enters Region II normal to the boundary

as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the

paper. The length of the Region II is l . Choose the correct choice(s).

Figure :



The particle enters Region III only if its velocity $V > \frac{qlB}{m}$

(A)

The particle enters Region III only if its velocity $V < \frac{qlB}{m}$

(B)

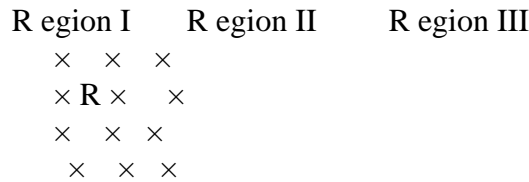
Path length of the particle in Region II is maximum when velocity $V = \frac{qlB}{m}$

(C)

Time spent in Region II is same for any velocity V as long as the particle returns to Region I

(D)

Sol. (A), (C), (D)



Radius of circular path inside region II is $R = \frac{mv}{qB}$

Time spent in region II as long as the particle returns to region I is

$$t = \frac{\pi R \sin \theta}{V} = \frac{\pi m}{qB}$$

particle enters region III if $R > l$

$$\Rightarrow V > \frac{qBl}{m} \quad \dots\dots\dots(i)$$

Maximum path length is $= \pi R = \pi l$

$$\therefore R = l \Rightarrow V = \frac{qBl}{m} \quad \dots\dots\dots(ii)$$

Time spent = $\frac{\pi m}{qB}$, which is independent of V.

31. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ . The intensity of light falling on slit 1 is four times the intensity of light falling on slit

2. Choose the correct choice(s).

(A) If $d = \lambda$, the screen will contain only one maximum
If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed

(B) on the screen

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

(D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

Sol. (A), (B)

$$\Delta x$$

No matter what the relation between λ and d be, the central bright fringe will always be formed.

Besides the central maximum, the first maximum will occur when the path difference $\Delta x = \lambda$.
If $d = \lambda$, $\Delta x = d \Rightarrow$ the maximum does not fall on the screen.

Hence, option (A) is correct.

$$d$$

$$(A) \rightarrow p_1 + p_2 = \frac{(a_1 + a_2)^2}{2} + \frac{(b_1 + b_2)^2}{2} + c_1 k \neq 0 \quad Q \quad c_1 \neq 0$$

$$(B) p_1 + p_2 = \frac{(c_1 + c_2)^2}{2} \quad k = 0 \text{ if } c_1 = -c_2$$

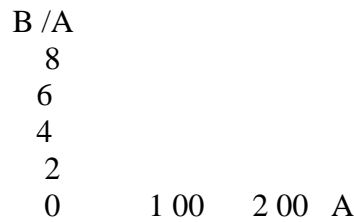
$$(C) p_1 + p_2 = \frac{(a_1 + a_2)^2}{2} + \frac{(b_1 + b_2)^2}{2} = 0 \text{ if } a_1 = -a_2 \text{ \& } b_1 = -b_2$$

$$(D) p_1 + p_2 = \frac{(a_1 + a_2)^2}{2} + 2b_1 \neq 0 \quad Q \quad b_1 \neq 0$$

33. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown

in the figure. Use this plot to choose the correct choice(s) given below.

Figure:



- (A) Fusion of nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy
- (B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy
- (C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments
- (D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments

Sol. (B), (D)

→ Mass number of the resultant nucleus will be < 100 . Thus, no change in B/A. Hence

no

- (A) energy released.
→ Mass number of the resultant nucleus will lie between 100 and 200. The B/A will
- (B) increases \Rightarrow resultant nucleus is more stable \Rightarrow energy is released.
→ The daughter nuclei will have $50 < A < 100 \Rightarrow$ B/A has decreased \Rightarrow stability has
- (C) decreased \Rightarrow no release of energy.
→ The daughter nuclei will have $100 < A < 130 \Rightarrow$ B/A has increased \Rightarrow stability has
- (D) increased \Rightarrow energy is released.

SECTION - III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

34. STATEMENT-1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT-2

Resistance of a metal increases with increase in temperature.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for

STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation

for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (C)

Resistance of a metal increases with increase in temperature. So statement 2 is true. If unknown

resistance increases, then the standard resistance must be increased to keep the ratio fixed (the null point remains the same).

SO STATEMENT 1 IS FALSE

35. STATEMENT-1

An astronaut in an orbiting space station above the Earth experiences weightlessness. and

STATEMENT-2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of

free-fall .

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for

STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation

for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 33

Sol. (A) In the frame of reference of the space station, the gravitational force balances the centrifugal

force and hence the normal reaction is zero.

Hence statement 1 is true.

As far as weightlessness is concerned above situation is equivalent to a state of free fall.

Hence 2 is true and an explanation for 1.

36. STATEMENT-1

The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain

when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT-2

In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for

STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation

for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (A)

This is a common observation.

Statement I is true.

$A_1 V_1 = A_2 V_2$ speed decreases as the fluid moves upward, due to conservation of energy.

Hence area of cross section increases.

Speed increases as the fluid moves down ward, due to conservation of energy y. Hence area of cross-section decreases.

Statement II is true and is a correct explanation of I

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 34

37. STATEMENT-1

Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical

dimensions are simultaneously allowed to roll without slipping down an inclined plane from the

same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT-2

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for

STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation

(C)

for STATEMENT-1

(D) STATEMENT-1 is True, STATEMENT-2 is False

(E) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (D)

If linear acceleration along the incline is a and length of the incline is

$2l$

$1/2$

$l, l, l = at \Rightarrow t =$

$2 \quad a$

$2l$

$=$

αr

$$= \frac{2I}{r} \times \frac{I}{mg \sin \theta}$$

Hence lower the moment of inertia, lower is the time taken. Hence solid cylinder will reach the bottom first.

Statement I is false

Using conservation of energy statement II is true

SECTION - IV

Linked Comprehension Type

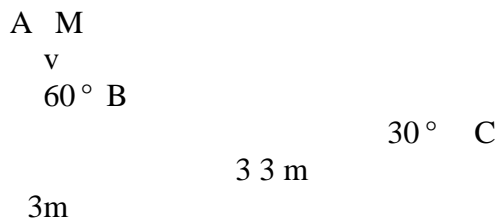
This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

Paragraph for Question Nos. 38 to 40

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure.

The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10 \text{ m/s}^2$).

Figure :



38. The speed of the block at point B immediately after it strikes the second incline is
 (A) 60 m/s (B) 45 m/s (C) 30 m/s (D) 15 m/s

Sol. (B)

Let V be the speed at point B just before it strikes the second incline.

$$\frac{1}{2} MV^2 = Mg \cdot 3 \tan 60^\circ$$

$$\therefore V = 6g \text{ m/s}$$

Let assume X axis along the incline at 30° and Y axis perpendicular to it.

$$\therefore \text{component of } V \text{ along X -direction } V_x = \frac{3V}{2} \cos 30^\circ = \frac{V}{2}$$

$$\text{Component of } V \text{ along Y -direction } V_y = \frac{3V}{2} \sin 30^\circ = \frac{3V}{4}$$

After the inelastic collision with second incline, Y-component will be zero and X-component will

remain unchanged.

∴ After collision velocity of the block, $V' = \frac{3V}{2}$ and is along X -direction

$$V' = \frac{3 \times 60}{2} = 45 \text{ m/s}$$

or,

39. The speed of the block at point C, immediately before it leaves the second incline is
 (A) 120 m/s (B) 105 m/s (C) 90 m/s (D) 75 m/s

Sol. (B)

From conservation of energy

$$Mg(3 + 3 \tan 30^\circ) = \frac{1}{2} M V_c^2 + \frac{1}{2} M V'^2$$

$$\Rightarrow V_c^2 = 6g + V'^2$$

$$= 60 + 45 = 105$$

$$V_c = 105$$

40. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

- (A) 30 m/s (B) 15 m/s (C) 0 (D) 15 m/s

Sol. (C)

If collision is perfectly elastic, x-and y-components of velocity of the block after collision, at B

$$V_{B,X} = V_{X'} = \frac{3V}{2} = 45 \text{ m/s}$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 37

$$V_{B,y} = -V_{Y'} = \frac{V}{2} = 15 \text{ m/s}$$

vertical component (upward) of the velocity of the block,

$$15 \cos 30^\circ - 45 \sin 30^\circ = 15 \cos 30^\circ - 45 \sin 30^\circ = 0$$

Paragraph for Question Nos. 41 to 43

In a mixture of H and He⁺ gas (He⁺ is singly ionized He atom), H atoms and He⁺ ions are excited to

their respective first excited states. Subsequently, H atoms transfer their total excitation energy

to

He⁺ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

The quantum number n of the state finally populated in He⁺ ions is

41.

- (A) 2 (B) 3 (C) 4 (D) 5

Sol. (C)

Initially,

$$E_{n=2, H} = -\frac{13.6}{2^2} = -13.6 \text{ eV}$$

$$E_{n=1, H} = -\frac{13.6}{1^2} = -13.6 \text{ eV}$$

E

Excitation energy of H = $E_{n=2, H} - E_{n=1, H}$

$$= -\frac{13.6}{4} - \left(-\frac{13.6}{1} \right) = 12.15 \text{ eV}$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 38

$$= \frac{3}{4} \times 13.6 \text{ eV}$$

Energy of He after collisions,

$$E_{n=3, He} = -\frac{13.6}{4} + \frac{3}{4} \times 13.6 \text{ eV}$$

$$= -\frac{13.6}{4} + \frac{39.6}{4} = \frac{26}{4} = 6.5 \text{ eV}$$

$$= -\frac{13.6}{4} + \frac{39.6}{4}$$

$$= -\frac{13.6}{4} + \frac{39.6}{4}$$

$$= E_{n=4, He}$$

The wavelength of light emitted in the visible region by He⁺ ions after collisions with H atoms is

42.

- (A) $6.5 \times 10^{-7} \text{ m}$ (B) $5.6 \times 10^{-7} \text{ m}$ (C) $4.8 \times 10^{-7} \text{ m}$ (D) $4.0 \times 10^{-7} \text{ m}$

Sol. (C)

Due to transition from n = 4 to lower orbits, H⁺ will emit radiations.

$$\left[\frac{1}{1} - \frac{1}{n^2} \right] hc$$

$$13.6 \times Z^2 \left(\frac{1}{n_2} - \frac{1}{n_1} \right) = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{13.6 \times 4^2 \left(\frac{1}{2} - \frac{1}{2} \right)}{1242 \times 10^{-9} \left(\frac{1}{n_1} - \frac{1}{n_2} \right)}$$

Putting transition from $n_2 = 4$ to $n_1 = 3$

$$= \frac{1242 \times 10^{-9} \times 22.8 \times 144}{22.8 \times 144 \times 10^{-9}} = 4.7 \times 10^{-7} \text{ m}$$

$$\lambda = \frac{13.6 \times 4^2 \left(\frac{1}{9} - \frac{1}{16} \right)}{1242 \times 10^{-9}}$$

So transition will lie in visible region.

For other transitions, λ is less than 300 nm.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 39

The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He^+ ion is

43.

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

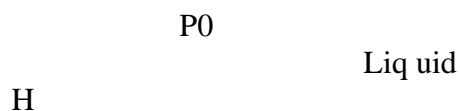
Sol. (C)

$$\frac{(K.E.)_{H,n=2}}{(K.E.)_{He,n=2}} = \frac{(T.E.)_{H,n=2}}{(T.E.)_{He,n=2}} = \frac{Z_H^2}{Z_{He}^2} = \frac{1^2}{4^2} = \frac{1}{4}$$

Paragraph for Question Nos. 44 to 46

A small spherical monoatomic ideal gas bubble of radius r is trapped inside a liquid of density ρ_l (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0 , the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure :



44. As the bubble moves upwards, besides the buoyancy force the following forces are acting on

it

- (A) Only the force of gravity
- (B) The force due to gravity and the force due to the pressure of the liquid
- (C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
- (D) The force due to gravity and the force due to viscosity of the liquid

Sol. (A)

Viscous force if present, will generate heat due to friction between bubble and liquid and hence heat exchange between the bubble & liquid contrary to what is given in the question. Hence no viscous forces are present.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 40

45. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0 \left(\frac{P_0 + \rho l g H}{P_0 + \rho l g y} \right)^{2/5}$

(B) $T_0 \left(\frac{P_0 + \rho l g (H - y)}{P_0 + \rho l g H} \right)^{2/5}$

(C) $T_0 \left(\frac{P_0 + \rho l g H}{P_0 + \rho l g y} \right)^{3/5}$

(D) $T_0 \left(\frac{P_0 + \rho l g (H - y)}{P_0 + \rho l g H} \right)^{3/5}$

$PV = nRT \rightarrow$ ideal gas equation.

Sol. (B)

$$\begin{aligned}
 & \text{H} \\
 & \text{y} \\
 & (P_0 + \rho l g H)V_0 = nRT_0 \\
 & PV = nRT \quad [P = P_0 + \rho l g (H - y)] \\
 & \text{also, } PV^\gamma = \text{const.} \\
 & \Rightarrow (P_0 + \rho l g H)V_0 = [P_0 + \rho l g (H - y)]V^\gamma \\
 & \Rightarrow V = \left[\frac{P_0 + \rho l g H}{P_0 + \rho l g (H - y)} \right]^{1/\gamma} V_0 \\
 & \Rightarrow T = \left(\frac{P_0 + \rho l g (H - y)}{P_0 + \rho l g H} \right)^{1-\gamma} T_0 \quad (5) \\
 & \quad \quad \quad (3) \\
 & \quad \quad \quad 1 \\
 & \quad \quad \quad 1- \\
 & \quad \quad \quad [\quad] \\
 & \quad \quad \quad P_0 + \rho l g H \gamma
 \end{aligned}$$

46. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

- (A) $\frac{\rho \ln R g T_0}{(P_0 + \rho l g H)^{2/5}}$
- (B) $\frac{(P_0 + \rho l g H) [P_0 + \rho l g(H - y)]^{3/5}}{2/5 \rho \ln R g T_0}$
- (C) $\frac{\rho \ln R g T_0}{(P_0 + \rho l g H)^{3/5}}$
- (D) $\frac{\rho \ln R g T_0}{(P_0 + \rho l g(H - y))^{8/5}}$
- (E) $\frac{(P_0 + \rho l g H) (P_0 + \rho l g(H - y))^{2/5}}{3/5}$

Sol. (B)

$$F_{\text{Buoyancy}} = \rho l V g = \rho g \left[\frac{(P_0 + \rho g H)^{1/\gamma}}{(P_0 + \rho g(H - y))^{1/\gamma}} \right] V_0$$

Using $V_0 =$

$$V_0 = \frac{1}{(P_0 + \rho g H)^{-1 + \frac{1}{\gamma}}}$$

$$F_B = \frac{\rho l g n R T_0 [P_0 + \rho g H]^{-1 + \frac{1}{\gamma}}}{[P_0 + \rho g(H - y)]^{1/\gamma}}$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 42

CHEMISTRY

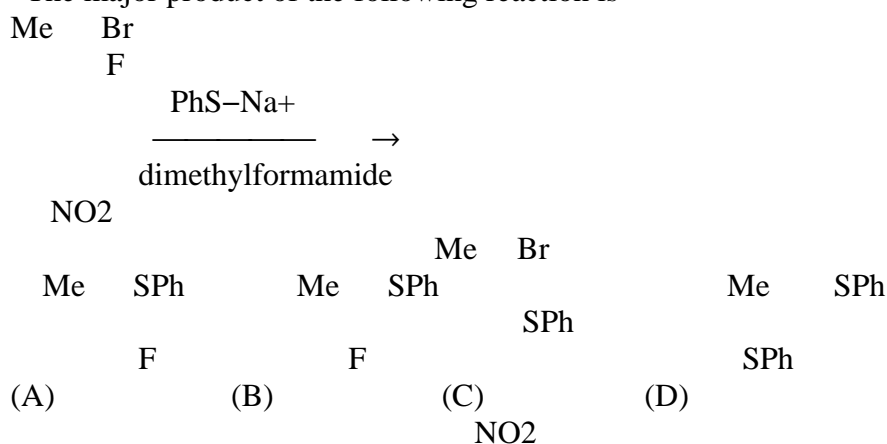
PART III

Section I

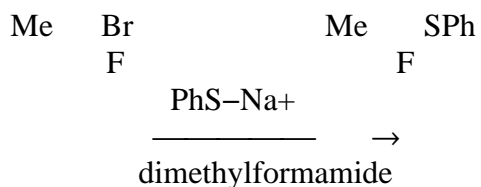
Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

47. The major product of the following reaction is

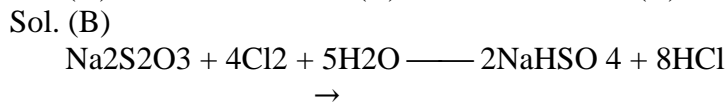


Sol. (A)



NO₂ NO₂
 SN₂ reaction takes place inversion takes place.

48. Aqueous solution of Na₂S₂O₃ on reaction with Cl₂ gives
 (A) Na₂S₄O₆ (B) NaHSO₄ (C) NaCl (D) NaOH



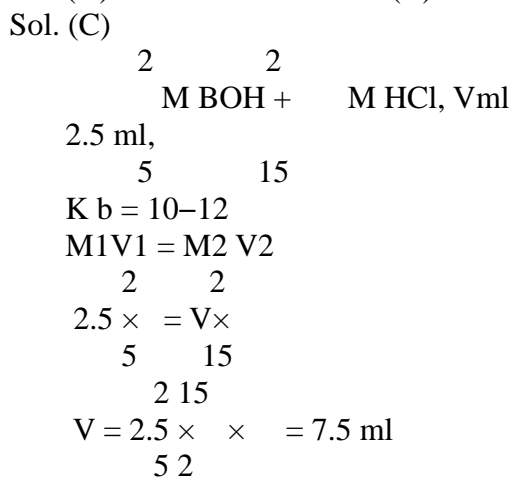
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49. Hyperconjugation involves overlap of the following orbitals
 (A) σ-σ (B) σ-p (C) p-p (D) π-π

Sol. (B)
 Hyperconjugation involves overlap of σ-p orbitals.

50. 2.5 mL of weak monoacidic base K_b = 1 × 10⁻¹² at 25 °C is titrated with M HCl in water

- (A) 3.7 × 10⁻¹³ M (B) 3.2 × 10⁻⁷ M (C) 3.2 × 10⁻² M (D) 2.7 × 10⁻² M



Number of moles of BCl formed = 1 × 10⁻³
 Total volume = 10 × 10⁻³ L

$$[\text{BCl}] = \frac{1 \times 10^{-3}}{10 \times 10^{-3}} = 0.1 \text{ M}$$

$$[\text{H}^+] = \frac{K_w}{K_b \cdot C} = \frac{10^{-14}}{0.1 \times 10^{-12}} = 10^{-3}$$

$$= 10^{-12} \\ = 10 \times 10^{-4} = 3.2 \times 10^{-2} \text{ M}$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 44

51. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the

presence of

- (A) nitrogen (B) oxygen (C) carbon dioxide (D) argon

Sol. (B)

Ag gets oxidised in presence of oxygen and dissolves forming the complex $\text{Na}[\text{Ag}(\text{CN})_2]$.

Under the same reaction conditions, initial concentration of $1.386 \text{ mol dm}^{-3}$ of a substance becomes

52.

half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively.

Ratio

$\left(\frac{k_1}{k_0} \right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reaction is

- (A) $0.5 \text{ mol}^{-1} \text{ dm}^3$ (B) 1.0 mol dm^{-3} (C) 1.5 mol dm^{-3} (D) $2.0 \text{ mol}^{-1} \text{ dm}^3$

Sol. (A)

$$[\text{A}]_i = 1.38 \text{ L mol}^{-1} \xrightarrow{\text{first order}} \frac{1.38}{2} \text{ in 40 sec.}$$

$$k_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{40}$$

$$[\text{A}]_f = \frac{1.386}{2} \text{ in 20 sec. for zero order}$$

$$t_{1/2} = 20 \text{ sec}$$

zero order

$$x = kt$$

a

$$= kt_{1/2}$$

2

$$k_0 = \frac{a}{2t_{1/2}} = \frac{1.38}{2 \times 20}$$

$$= \frac{0.693}{40}$$

$$\Rightarrow \frac{k_1}{k_0} = \frac{0.693}{0.693} = 1.0$$

$$k_1 = \frac{0.693}{40}$$

$$= 40 = 0.502$$

$$\Rightarrow \frac{k_1}{k_0} = 0.502$$

$$k_0 = \frac{1.38}{40}$$

$$= \frac{1.38}{40}$$

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which one or more is/are correct.

53. A solution of colourless salt H on boiling with excess NaOH produces a non-flammable gas. The

gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt(s) H is (are)

- (A) NH_4NO_3 (B) NH_4NO_2 (C) NH_4Cl (D) $(\text{NH}_4)_2\text{SO}_4$

Sol. (A, B)

On addition of NaOH all ammonium salts give ammonia, but since the evolution of NH_3 resumes

on addition of Zn, the salts can be NO_3 or NO_2 which on reduction give ammonia.

54. A gas described by van der Waals equation

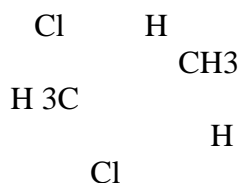
- (A) behaves similar to an ideal gas in the limit of large molar volumes
 (B) behaves similar to an ideal gas in the limit of large pressures
 (C) is characterised by van der Waals coefficients that are dependent on the identity of the gas

but

- are independent of the temperature
 (D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally.

Sol. (A, C, D)

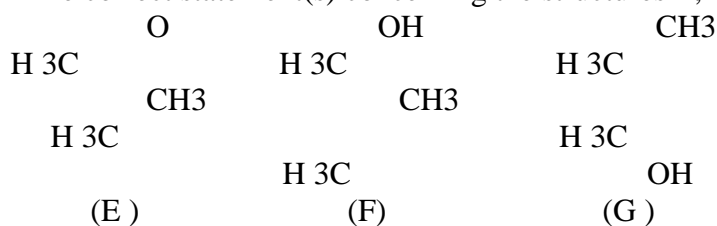
55. The correct statement(s) about the compound given below is (are)



- (A) The compound is optically active
 (B) The compound possesses centre of symmetry
 (C) The compound possesses plane of symmetry
 (D) The compound possesses axis of symmetry

Sol. (C, D)

56. The correct statement(s) concerning the structures E, F and G is (are)



- (A) E, F and G are resonance structure (B) E, F and G are tautomers
 (C) F and G are geometrical isomers (D) F and G are diastereomers

Sol. (B, C, D)

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 46

Section III
Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

57. Statement 1: Bromobenzene upon reaction with Br_2/Fe gives 1,4-dibromobenzene as the major

product

Statement 2: In bromobenzene, the inductive effect of the bromo group is more dominant than the

mesomeric effect in directing the incoming electrophile.

(A) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement

1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

Sol. (A)

Br

Br

Br

Br

Br_2, Fe

+

(minor product)

Br

(major product)

The ortho, para - directive influence of Br is due to + mesomeric effect.

Statement 1: Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.

58.

Statement 2: The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to inert pair effect .

(A) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement

1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

Sol. (C)

Pb^{4+} is less stable than Pb^{2+} where Sn^{4+} is more stable than Sn^{2+} , Pb^{4+} , therefore is a stronger

oxidizing agent.

The higher oxidation states of heavier elements are less stable due to inert pair effect.

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 47

59. Statement 1: For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

Statement 2: At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

(A) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement

1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

Sol. (D)

For a chemical reaction at equilibrium

$\Delta G = 0, \Delta G^\circ \neq 0$

For a spontaneous process

$$\Delta G = -ve$$

60. Statement 1: The plot of atomic number (y-axis) versus number of neutrons (x-axis) for stable nuclei shows a curvature towards x-axis from the line of 45° slope as the atomic number is increased.

Statement 2: Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons in heavier nuclides.

(A) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

Sol. (B)

Both statements are correct but statement 2 is not explanation of statement 1.

z

Stable
nuclei
45°
n

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 48

Section IV Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be

answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Paragraph for Question Nos. 61 to 63

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when solute

molecules are added to get homogeneous solution. These are called colligative properties.

Applications

of colligative properties are very useful in day-to-day life. One of its examples is the use of ethylene glycol

and water mixture as anti-freezing liquid in the radiator of automobiles

A solution M is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture is 0.9

()

water

$$= 1.86 \text{ K kg mol}^{-1}$$

Freezing point depression constant of water K_f

Given :

()

ethanol

$$= 2.0 \text{ K kg mol}^{-1}$$

Freezing point depression constant of ethanol K_f

()

water

$$= 0.52 \text{ K kg mol}^{-1}$$

Boiling point elevation constant of water K_b

()

$$\text{ethanol} = 1.2 \text{ K kg mol}^{-1}$$

Boiling point elevation constant of ethanol K_b

Standard freezing point of water = 273 K

Standard freezing point of ethanol = 155.7 K

Standard boiling point of water = 373 K

Standard boiling point of ethanol = 351.5 K

Vapour pressure of pure water = 32.8 mm Hg

Vapour pressure of pure ethanol = 40 mm Hg

Molecular weight of water = 18 g mol⁻¹

Molecular weight of ethanol = 46 g mol⁻¹

In answering the following questions, consider the solutions to be ideal dilute solutions and solutes to be

non-volatile and non-dissociative.

61. The freezing point of the solution M is

- (A) 268.7 K (B) 268.5 K (C) 234.2 K (D) 150.9 K

Sol. (D)

Let total moles be 1, then

$$n_{\text{ethanol}} = 0.9$$

$$\text{mass of ethanol} = 41.4 \text{ g} = 41.4 \times 10^{-3} \text{ kg}$$

$$n_{\text{water}} = 0.1$$

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 49

$$0.1$$

$$\times 10^3 = 2.4 \text{ mol/kg.}$$

molality, $m =$

$$\frac{2.4}{41.4}$$

$$\Delta T_f = K_f m = 2 \times 2.4 = 4.8$$

$$T_{f, \text{ethanol}} = 155.7 - 4.8 = 150.9 \text{ K}$$

62. The vapour pressure of the solution M is

- (A) 39.3 mm Hg (B) 36.0 mm Hg (C) 29.5 mm Hg (D) 28.8 mm Hg

Sol. (B)

$$40 - P_s$$

$$= 0.1$$

$$\frac{40 - P_s}{40}$$

$$P_s = 36 \text{ mm of Hg.}$$

63. Water is added to the solution M such that the mole fraction of water in the solution becomes 0.9.

The boiling point of this solution is

- (A) 380.4 K (B) 376.2 K (C) 375.5 K (D) 354.7 K

Sol. (B)

Let no of moles be 1

$$n_{\text{H}_2\text{O}} = 0.9, n_{\text{ethanol}} = 0.1$$

$$\text{mass of H}_2\text{O} = 0.9 \times 18 \text{ g} = 16.2 \text{ g} = 0.0162 \text{ kg}$$

$$0.1$$

$$\text{molality} = \frac{0.1}{0.0162} \text{ mol / kg} = 6.17 \text{ mol/kg}$$

$$0.0162$$

$$\Delta T_b = k_b \cdot m$$

$$= 0.52 \times 6.17 = 3.2$$

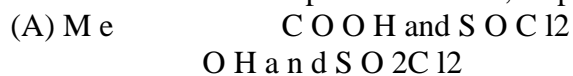
$$\text{Boiling point of solution} = 373 + 3.2 = 376.2 \text{ K}$$

Paragraph for Question Nos. 64 to 66

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water.

- (C) Me $\begin{matrix} \text{Br} \\ | \\ \text{C} \\ | \\ \text{Br} \end{matrix}$
 (D) Me $\begin{matrix} \text{Br} \\ | \\ \text{C} \\ | \\ \text{Br} \end{matrix}$
 Sol. (D)

68. The structures of compounds J and K, respectively, are



(B) Me



Sol. (A)

69. The structure of product L is



(C) Me



Sol. (C)

IIT-JEE 2008 SOLUTIONS PAPER 1

Page 52

Explanation for questions 67 to 69:

