MATHEMATICS

PART I

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) , out of which **ONLY ONE** is correct.

1. Let g(x) = log f(x) where f(x) is a twice differentiable positive function on $(0,\infty)$ such that f(x + 1) = x f(x). Then, for N = 1, 2, 3,...,

(A) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N-1)^2}\right\}$ (B) $4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N+1)^2}\right\}$ (C) $4\left\{1+\frac{1}{9}+\frac{1}{25}+...+\frac{1}{(2N+1)^2}\right\}$

Sol. (A) g(x) = log f(x)

$\Rightarrow \quad g(x+N)=\log f(x+N)$

$f(x+1)=x\,f(x)$

$\therefore \quad f(x+N)=(x+N-1)f(x+N-1)$

$=(x+N-1)(x+N-2)f(x+N-2)...$

$=(x+N-1)(x+N-2).........(x+1)xf(x)$

$\log f(x+N) \quad =\log(x+N-1)+\log(x+N-2)+...\log(x+1)+\log x+\log f(x)$

$\log f(x+N)-\log f(x)=\log x+\log(x+1)+...\log(x+N-1)$

$g(x+N)-g(x)=\log x+\log(x+1)+...+\log(x+N-1)$

$$g'(x+N)-g'(x)=\frac{1}{x}+\frac{1}{x+1}+...+\frac{1}{x+N-1}$$

$$g''(x+N)-g''(x)=-\left[\frac{1}{x^2}+\frac{1}{(x+1)^2}+...\frac{1}{(x+N-1)^2}\right]$$

putting x = $\frac{1}{2}$

$$g''\left(N+\frac{1}{2}\right)=g''\left[\frac{1}{2}\right]=-\left[\frac{1}{\left(\frac{1}{2}\right)^2}+\frac{1}{\left(\frac{3}{2}\right)^2}+...\frac{1}{\left(\frac{2N-1}{2}\right)^2}\right]=-4\left[1+\frac{1}{9}+...\frac{1}{(2n-1)^2}\right]$$

2. Let non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}$ cost + $\hat{b}$ sin t. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{\frac{1}{2}}$

(B) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{\frac{1}{2}}$

(C) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{\frac{1}{2}}$

(C) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{\frac{1}{2}}$

Sol. (A) $\hat{u}$ is along internal angle bisector of $\hat{a}$ & $\hat{b}$

$$\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$$

$$\overrightarrow{OP} = \hat{a}\cos t + \hat{b}\sin t$$

$$|\overrightarrow{OP}|^2 = 1 + 2\hat{a}\cdot\hat{b}\sin 2t$$

$$= 1 + \hat{a}\cdot\hat{b}\sin 2t$$

$$M^2 = 1 + \hat{a}\cdot\hat{b}$$

$$M = (1+\hat{a}\cdot\hat{b})^{\frac{1}{2}}$$

3. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1}dx,\ J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx.$

The, for an arbitrary constant C, the value of J – I equals

(A) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$

(B) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(C) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(C) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

Sol. (C) $J - I = \int\left(\frac{e^{-x}}{e^{-4x}+e^{-2x}+1} - \frac{e^{x}}{e^{4x}+e^{2x}+1}\right)dx$

$$\Rightarrow J - I = \int \left(\frac{e^{3x} - e^{x}}{e^{4x} + e^{2x} + 1} \right) dx$$

$$\Rightarrow J - I = \int \frac{e^{2x}(e^{x} - e^{-x})}{(e^{4x} + e^{2x} + 1)} dx$$

$$J - I = \int \frac{(e^{x} - e^{-x})}{e^{2x} + 1 + e^{-2x})} dx$$

Put $e^x = t \Rightarrow dx = \frac{dt}{e^x}$ or $\frac{dt}{t}$

$$\Rightarrow J - I = \int \frac{(t - 1/t)}{(t^2 + 1 + 1/t)} \frac{dt}{t}$$

$$= \int \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t^2 + \frac{1}{t^2} + 1\right)} dt$$

$$J - I = \int \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t}\right)^2 - 1} dt$$

Put $t + \frac{1}{t} = u \quad \Rightarrow \left(1 - \frac{1}{t^2}\right) dt = du$

$$\Rightarrow \quad J - I = \int \frac{du}{u^2 - 1}$$

$$\Rightarrow \quad J - I = \frac{1}{2}\int \frac{du}{u - 1} - \frac{1}{2}\int \frac{du}{u + 1}$$

$$\therefore \quad J - I = \frac{1}{2} \log \frac{u - 1}{u + 1}$$

$$\therefore \quad J - I = \frac{1}{2} \log \frac{t + \frac{1}{t} - 1}{t + \frac{1}{t} + 1} \qquad \text{as } u = t + \frac{1}{t}$$

$$\therefore \quad J - I = \frac{1}{2}\log\left(\frac{t^2 - t + 1}{t^2 + t + 1}\right)$$

$$\therefore \quad J - I = \frac{1}{2}\log\left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1}\right) \qquad \text{as } t = e^x$$

4. Consider three points. $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,

(A) P lies on the line segment RQ
(B) Q lies on the line segment PR
(C) R lies on the line segment QP
(D) P,Q,R are non-collinear

Sol. (D) Let $\alpha = \beta = \theta = \pi/6$

$$\therefore \; P = \left(O; -\frac{\sqrt{3}}{2}\right), Q = \left(1, \frac{1}{2}\right) \; \sqrt{R} = \left(\frac{\sqrt{3}}{2}, O\right)$$

which are non-collinear therefore option (D) is correct

5. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8
(B) 3, 6 or 9
(c) 4 or 8
(d) 5 or 10

Sol. (D) By using option (D)

$$P(A \cap B) = \frac{4}{10} \times \frac{5}{10} = \frac{2}{10}$$

$$P(A \cap B) = \frac{4}{10} \times \frac{10}{10} = \frac{4}{10}$$

hence option (D) is correct

6. The area of the region between the curves $y = \sqrt{\dfrac{1+\sin x}{\cos x}}$ and $y = \sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \dfrac{\pi}{4}$ is

(A) $\displaystyle\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(B) $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(C) $\displaystyle\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(D) $\displaystyle\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

Sol. **(B)**

Desired Area $\displaystyle = \int_0^{\pi/4} \left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}} \right) dx$

$$= \int_0^{\pi/4} \left(\sqrt{\frac{1+\dfrac{2\tan x/2}{1+\tan^2 x/2}}{\dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}}} - \sqrt{\frac{1-\dfrac{2\tan x/2}{1+\tan^2 x/2}}{\dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}}} \right) dx$$

$$= \int_0^{\pi/4} \left(\frac{|(\tan x/2+1)|}{\sqrt{1-\tan^2 x/2}} - \frac{|(1-\tan x/2)|}{\sqrt{1-\tan^2 x/2}} \right) dx$$

$$= \int_0^{\pi/4} \frac{2\tan x/2}{\sqrt{1-\tan^2 x/2}}\,dx \qquad [\text{as } 0 < x < \pi/4]$$

put $\tan x/2 = t$

$dx = \dfrac{2dt}{(1+t^2)}$

Area $\displaystyle = \int_0^{\sqrt{2}-1} \frac{2t}{\sqrt{1-t^2}} \cdot \frac{2dt}{(1+t^2)}$

$$= \int_0^{\sqrt{2}-1} \frac{4t}{\sqrt{1+t^2}\sqrt{1-t^2}}\,dt$$

7. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1-\sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}}-1$ (c) $1+\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}+1$

Sol. **(B)** $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$

$$\Rightarrow \frac{(x-\sqrt{2})^2}{4} - \frac{(y-\sqrt{2})^2}{2} = 1$$

$$\Rightarrow \frac{X^2}{2^2} - \frac{y^2}{2} = 1$$

$$e = \sqrt{1+\frac{2}{4}} = \sqrt{\frac{3}{2}}$$

Hence required Area $= \frac{1}{2}(ae - a)\frac{b^2}{a}$

$$= \frac{1}{2} \times 2\left(\frac{\sqrt{3}}{\sqrt{2}} - 1\right) \times \frac{2}{2}$$

$$= \sqrt{\frac{3}{2}} - 1$$

Hence option (B) is correct

8. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(A) 6 + 7i (B) – 7 + 6i (c) 7 + 6i (d) – 6 + 7i

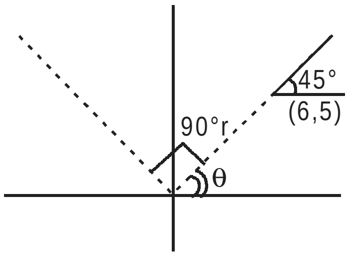
Sol. **(D)** $z_0(1,2)$

$z_1(6,5)$

$r\cos\theta = 7$

$r\sin\theta = 6$

$Z_2(r\cos(\theta+90°), r\sin(\theta+90°))$



$Z_2(r\cos(\theta+90°), r\sin(\theta+90°)$

$\Rightarrow \quad Z_2(-r\sin\theta, r\cos\theta)$

$\Rightarrow \quad Z_2(-6, 7)$

$\therefore \quad Z_2 = -6+7i$

9. Let the function $g:(-\infty,\infty) \to \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is

(A) even and is strictly increasing in $(0,\infty)$

(B) odd and is strictly decreasing in $(-\infty,\infty)$

(C) odd and is strictly increasing in $(-\infty,\infty)$

(D) neither even nor odd, but is strictly increasing in $(-\infty,\infty)$

Sol. (C)

$$g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$$

$$\therefore \quad g(u) = 2\frac{1}{(1+e^{2u})}e^u$$

$$\therefore \quad g'(u) > 0$$

Hence, monotonically increasing in $(-\infty,\infty)$

Now $\quad g(-u) = 2\tan^{-1}(e^{(-u)}) - \frac{\pi}{2}$

$$= 2\tan^{-1}\left(\frac{1}{e^u}\right) - \frac{\pi}{2}$$

$$= 2\cot^{-1}\left(e^u\right) - \frac{\pi}{2}$$

$$= 2\left(\frac{\pi}{2} - \tan^{-1}(e^u)\right) - \frac{\pi}{2}$$

$$= \pi - 2\tan^{-1}e^u - \frac{\pi}{2}$$

$$= -2\tan^{-1}e^u + \frac{\pi}{2}$$

$$= -g(u)$$

As $g(-u) = -g(u)$

Hence g(u) is odd function

SECTION - II

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

10. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{\beta}{2}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$.

STATEMENT-1 : $(p^2 - q)(b^2 - ac) \ge 0$

and

STATEMENT-2 : $b \ne pa$ or $c \ne qa$

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (A) According to condition $\beta^2 \notin \{1, 0, -1\}$ α, β both real

$\Rightarrow D_1 \ge 0, D_2 \ge 0$

$\Rightarrow D_1 \times D_2 \ge 0$

& $b \neq pa, c \neq qa$

11. Consider

$L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0$

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0.$

STATEMENT-1 : In line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

STATEMENT-2 : In line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (C) If L_1 is the diameter then p = – 6

L_2 becomes 2x + 3y – 3 = 0

Now distance of (–3, 5) From L_2 is

$$\left|\frac{-6+15-3}{\sqrt{13}}\right| = \frac{6}{\sqrt{13}} < 2$$

$\Rightarrow$ L_2 is chord

12. Let a solution y = y(x) of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.

STATEMENT-1 : $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$

and

STATEMENT-2 : y(x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (C) $\frac{1}{y} = \cos\left(\sec^{-1} x - \frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2x} + \frac{1}{2}\sqrt{1 - \frac{1}{x^2}}$

13. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

STATEMENT-2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (C) Let $a_1 = 1, a_2 = 2, a_3 = 4, a_4 = 8$

$b_1 = 1, b_2 = 3, b_3 = 7, b_4 = 15$

SECTION - III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos.14 to 16.

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

14. The unit vector perpendicular to both L_1 and L_2 is

(A) $\dfrac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$

(B) $\dfrac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(C) $\dfrac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(D) $\dfrac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

Sol. (B) $\vec{b}_1 = 3\hat{i}+\hat{j}+2\hat{k}$

$\vec{b}_2 = \hat{i}+2\hat{j}+3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \hat{i}(3-4) - \hat{j}(9-2) + \hat{k}(6-1)$$

$\therefore$ Unit vector $= \dfrac{-\hat{i}-7\hat{j}+5\hat{k}}{\sqrt{1+49+25}}$

$$\frac{-\hat{i}-7\hat{j}+5\hat{k}}{\sqrt{75}}$$

15. The shortest distance between L_1 and L_2 is

(A) 0

(B) $\frac{17}{\sqrt{3}}$

(C) $\frac{41}{5\sqrt{3}}$

(D) $\frac{17}{5\sqrt{3}}$

Sol. (D) $\vec{a}_2 - \vec{a}_1(1+2)\hat{i} + (2-2)\hat{j} + (1+31\hat{x})$

$= 3\hat{i} + o\hat{j} + 4\hat{u}$

$(a_2 - a_1)\cdot(a_1 \times a_2)\begin{vmatrix} 3 & 0 & 4 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$3(3-4)+4(6-1)$

$-3 + 20 = 17$

S.D = $\frac{17}{\sqrt{75}}$

16. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$

(B) $\frac{7}{\sqrt{75}}$

(C) $\frac{13}{\sqrt{75}}$

(D) $\frac{23}{\sqrt{75}}$

Sol. (C) $a(x+1)+b(y+2)+c(2+1)=0$

$\therefore$ O Normal of plane is perpendicular to line

$\Rightarrow \begin{matrix} 3a+b+2c=0 \\ a+2b+3c=0 \end{matrix}$

$\frac{a}{3-4} = \frac{b}{2-9} = \frac{c}{6-1}$

$\frac{a}{-1} = \frac{b}{-7} = \frac{c}{5}$

$\therefore$ Equation of plane is

$-(x+1)-7(y+2)+5(z+1)=0$

$-x-1-7y-14+5z+5=0$

$-x-7y+5z-10=0$

$$\frac{|-1-7+5-10|}{\sqrt{1+49+25}} = \frac{13}{\sqrt{75}}$$

Paragraph for Question Nos. 17 to 19

Consider the function $f:(-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \ 0 < a < 2$$

17. Which of the following is true?

(A) $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$

(B) $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$

(C) $f'(1)f'(-1) = (2-a)^2$

(D) $f'(1)f'(-1) = -(2+a)^2$

Sol. (A) $f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \ 0 < a < 2$

$$f'(x) = \frac{(2x-a)(x^2+ax+1) - (2x+a)(x^2-ax+1)}{(x^2+ax+1)^2}$$

$$f'(x) = \frac{2a(x^2-1)}{(x^2+ax+1)} \quad f'(1) = f'(-1) = 0$$

$$f'(x)(x^2+ax+1)^2 - 2a(x^2-1) = 0$$

$$f''(x)(x^2+ax+1)^2 + f'(x)(x^2+ax+1)(2x+a) - 4ax = 0$$

$$f''(1)(2+a)^2 = 4a$$

$$f''(-1)(2-a)^2 = -4a$$

$$f''(1)(2+a)^2 + f(-1)(2-a)^2 = 0$$

$\therefore$ **(A) is correct**

18. Which of the following is true?

(A) f(x) is decreasing on (–1, 1) and has a local minimum at x = 1

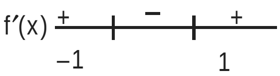
(B) f(x) is increasing on (–1, 1) and has a local maximum at x = 1

(C) f(x) is increasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1

(D) f(x) is decreasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1

Sol. (A) $f(x) = \frac{2a(x^2-1)}{(x^2+ax+1)^2}$

$\therefore\ f'(x) < 0, x \in (1,1)$



x = 1, Minima hence option (A) is correct

19. Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$

Which of the following is true?

(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$

(B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$

(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$

(D) $g'(x)$ does not changes sign on $(-\infty, \infty)$

Sol. (B) $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$

$$g'(x) = e^x \cdot \frac{f'(e^x)}{1+c^{2x}}$$

$$= e^x \cdot \frac{2a(e^{2x}-1)}{1+e^{2x}}$$

when $x > 0, \quad g'(x) > 0$

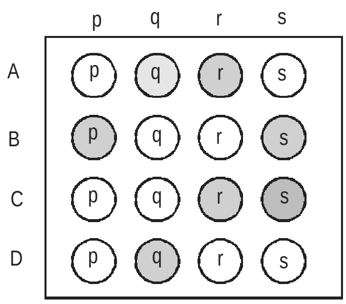
$x < 0, \quad g'(x) < 0$

SECTION - IV

Matrix Match Type

The section contains 3 questions. Each question contains statements given in two columns, which have to be matched. Statements in **Column I** are labelled as A, B, C and D whereas statements in **Columns II** are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-q, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following:



20. Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the approprite bubbles in the 4 × 4 matrix given in the ORS.

Column II	**Column II**
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(p) 0
(B) Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is key-symmetric, and $(A+B)(A-B)=(A-B)(A+B)$. If $(AB)^t=(-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(C) Let $a=\log_3\log_3 2$. An integer k satisfying $1<2^{\left(-k+3^{-a}\right)}<2$, must be less than	(r) 2
(D) If $\sin\theta=\cos\phi$, then the possible value of $\frac{1}{\pi}\left(\theta\pm\varphi-\frac{\pi}{2}\right)$ are	(s) 3

Sol. $A\to r, B\to q,s, C\to r,s, D\to p,r$

(A) $f(x)=\frac{x^2+2x4}{x+2}$

$f'(x) = 0 \rightarrow x = 0, -4$

x = 0 is local minima

$f(0) = 2$

(B) q,s

$A' = A, B' = -B, AB = BA$

$(AB)^t = B^t A^t = -BA = (-1)^k AB$

$\Rightarrow (-1)^K = -1 \Rightarrow K = 1, 3$

(C) r,s

$2^0 < 2^{-k+3^{-a}} < 2^1$

$0 < -k + \log_2 3 < 1$

$\log_2 3 - 1 < k < \log_z 3 \Rightarrow k = 1$

(D) p,

$r \sin\theta = \cos\phi \Rightarrow \frac{\pi}{2} - \theta = 2n\pi \pm \phi \Rightarrow \frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) = -2n$

21. Consider all possible permutations of the letters of the word ENDEANOEL Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the approprite bubbles in the 4 × 4 matrix given in the ORS.

	Column II	**Column II**
(A)	The number of permutations containing the world ENDDEA is	(p) 5 !
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	(q) 2 × 5 !
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(r) 7 × 5 !
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s) 21 × 5 !

Sol. A - p; B - s; C - q; D - q

ENDEANOEL has $3E'S, 2N'S$.

(A) The number of permutations containing the word ENDEA?

Consider ENDEA as a group. There are 4 other different letters. Hence total number of permutation = 5! (ie P)

(B) The first and last letters are E.

The Number of ways of permutes the letters from 2nd to 8th position is $\frac{7!}{2!} = 21 \times 5!$. (ie. s)

(C) The letters that cannot be in the last 5 positions are are D,L,N.

Hence they have to be arranged in the 1st four position and the rest in the last 5 position. This can be done in

$\left(\frac{4!}{2!}\right)\times\left(\frac{5!}{3!}\right)=2\times5!$ (ie q)

(D) The odd position are 1, 3, 5, 7 and q. A, E, O are only in odd position and the rest in even positions.

This can be done in $\left(\frac{5!}{3!}\right)\times\frac{4!}{2!}=2\times5!$ (ie q)

22. Consider the lines given by

$$L_1 : x+3y-5=0$$
$$L_2 : 3x-ky-1=0$$
$$L_3 : 5x+2y-12=0$$

Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the approprite bubbles in the 4 × 4 matrix given in the ORS.

	Column II	**Column II**
(A)	L_1, L_2, L_3 are concurrent , if	(p) k = –9
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k=-\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(r) $k=\frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(s) k = 5

Sol. $L_1 : x+3y-5=0$

$L_2 : 3x-ky-1=0$

$L_3 : 5x+2y-12=0$

L_1, L_3 interest at (2, 1)

(A) For L_1, L_2, L_3 to be concurrent $3x-ky-1=0$

(S) must pass through (2, 1). Hence K = 5 (ie s)

(B) One of L_1, L_2, L_3 to parallel to atleast one of the other two

L_1, L_3 are not parallel.

So L_2 has to be either parallel to L_1 or to L_3.

Slope of $L_1 = -\frac{1}{3}$; $L_3=-\frac{5}{2}$

Hence Slope of $L_2 = -\frac{1}{3}$ if $\boxed{K = -9} \longrightarrow$ P or slope of $L_2 = -\frac{5}{2}$ if $\boxed{K = -\frac{6}{5}} \longrightarrow$ q

(C) L_1, L_2, L_3 to form a triangle they must not be concurrent or L2 cannot be parallel to either L_1 or L_3.

Hence only possibility is $\boxed{K = -\frac{6}{5}} \longrightarrow$ r

(D) L_1, L_2, L_3 cannot form a triangle if K = 5 or – 9 or $-\frac{6}{5}$ P, q, or S

PHYSICS

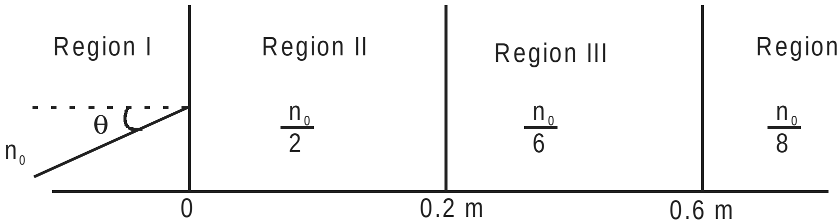
PART II

SECTIONS – I

Straight Objective Type

This section contains 9 multiple choice questions. Each questions has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

23. A light beam is traveling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are $n_0, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is
Figure:



(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{8}\right)$ (C) $\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

Sol. (B)

If the angle of incidence between Regions III and IV be ϕ, then

$$\frac{n_0}{6}\sin\phi = \frac{n_0}{8}\sin 90^\circ$$

$$\Rightarrow \sin\phi = \frac{3}{4}$$

Let the angle of incidence between Regions II and III be α. Then

$$\frac{n_0}{2}\sin\alpha = \frac{n_0}{6}\sin\phi$$

$$\Rightarrow \sin\alpha = \frac{\sin\phi}{3}$$

But $n_0 \sin\theta = \frac{n_0}{2}\sin\alpha$

$\therefore \sin\theta = \frac{\sin\alpha}{2} = \frac{\sin\phi}{6} = \frac{1}{8}$

$\therefore \theta = \sin^{-1}\left(\frac{1}{8}\right)$

24. A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s the frequency n of the tuning fork in Hz is

(a) 344 (B) 336 (C) 117.3 (D) 109.3

Sol. (A)

Third harmonic in closed pipe:

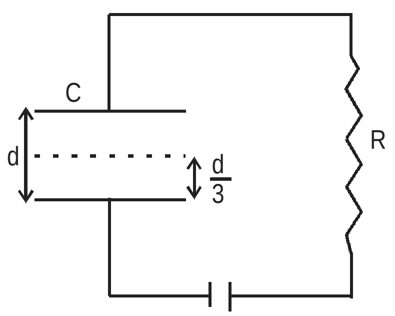
$$f = \frac{3v}{4L} = \frac{3\times 340}{4\times 0.75} = 340 \text{ Hz}$$

The string has the same frequency on increasing T, f increases and the number of beats with n decreases. Hence n – 340 = 4

$\therefore$ n = 344 Hz

25. A parallel plate capacitor C with plates of unit area and separation s is filled with a liquid of dielectric constant K = 2. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is

Figure



(A) $\frac{6\varepsilon_0 R}{5d + 3Vt}$ (B) $\frac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(C) $\frac{6\varepsilon_0 R}{5d - 3Vt}$ (D) $\frac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

Sol. (A)

The capacitance is given by

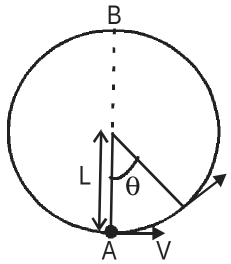
$$C = \frac{t_0}{\left[d-\left(\frac{d}{3}-Vt\right)\right]+\frac{1}{2}\left(\frac{d}{3}-Vt\right)}$$

$$= \frac{6\varepsilon_0}{5d+3Vt}$$

Time constant = RC = $\frac{6\varepsilon_0 R}{5d+3Vt}$

26. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies

Figure :



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ (C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

Sol. (D)

$V = \sqrt{5gL}$

Since mechanical energy is conserved,

$$\frac{1}{2}m(5gL) = \frac{1}{2}m\left(\frac{5gL}{4}\right)+mgh$$

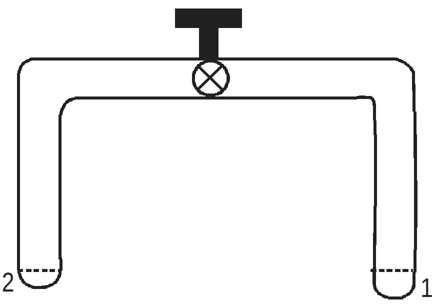
$$\Rightarrow h = \frac{15L}{8} = L + \frac{7L}{8}$$

$$\Rightarrow L(1-\cos\theta) = L\left(1+\frac{7}{8}\right)$$

Clearly, $-1 < \cos\theta < -\frac{1}{\sqrt{2}}$ $\Rightarrow \frac{3\pi}{4} < \theta < \pi$

27. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve,

Figure :



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles

(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) no change occurs

(D) air from end 2 flows towards end 1. Volume of hte soap bubble at end 1 increases

Sol. (B)

Let the radius of the bubble at end 2 be R. Then R > r.

Now, $P_2 - P_0 = \frac{4T}{R}$

$\Rightarrow P_2 = P_0 + \frac{4T}{R}$

And, $P_1 - P_0 = \frac{4T}{r}$

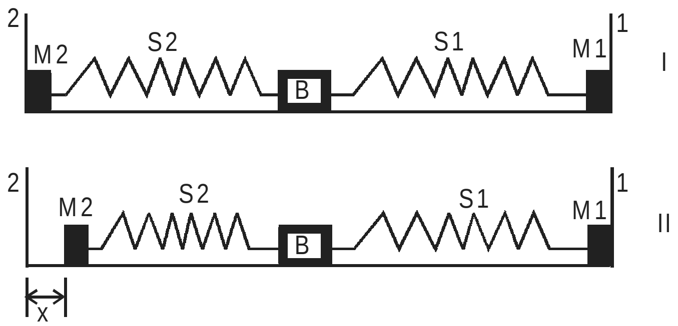
$\Rightarrow P_1 = P_0 + \frac{4T}{r}$

$\therefore P_1 > P_2$

$\Rightarrow$ Air will flow from end 1 to 2, and as a result volume at end 1 decreases.

28. A block (B) attached to two unstretched springs S1 and S2 with spring constants k and 4k, respectively (see figure 1). The other ends are attached to identical supports M1 and M2 not attached to hte walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to hte equilibrium position of the block B. The ratio $\frac{y}{x}$ is

Figure :



(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Sol. (C)

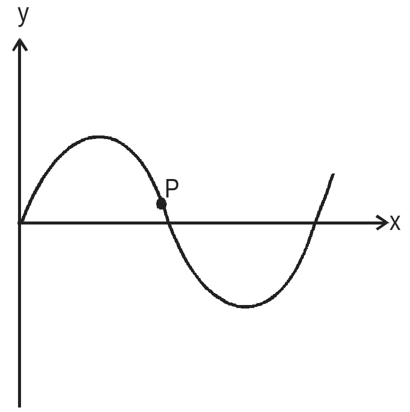
Energy of the system (B + springs) will be conserved.

$$\therefore \frac{1}{2}Kx^2 = \frac{1}{2}(4K)y^2$$

$$x = 2y$$

$$\therefore \frac{y}{x} = \frac{1}{2}$$

29. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snapshot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is Figure :



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$ (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$ (C) $\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$ (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$

Sol. (A)

$K = \frac{2\pi}{\lambda} = 4\pi\ rad/m$

$\omega = Kv = 4\pi.0.1 = 0.4\pi\ rad/s$

$\therefore y = 0.1\sin(4\pi x - 0.4\pi t)$

Suppose the snapshot shown is at t = 0.

$\therefore y = 0.1\sin(4\pi x)$

For P,

$0.05 = 0.1\ \sin(4\pi x)$

$\therefore \sin 4\pi x = 0.5$

Now velocity of P:

$v = \frac{dy}{dt} = -0.04\pi\cos(4\pi x - 0.4\pi t)$

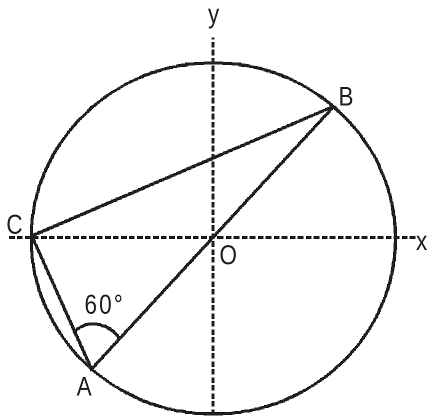
At t = 0,

$v = -0.04\pi\cos 4\pi x$

$= -0.04\pi\left(\pm\sqrt{(1-0.25)}\right) = \frac{\pi\sqrt{3}}{50} \quad \left(\because\ x > \frac{\pi}{2}\right)$

30. Consider a system of three charges $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60°

Figure :



(A) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis

(B) The potential energy of the system is zero

(C) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$

(D) The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

Sol. (C)

Electric field at O:

$$E = \frac{2q/3}{4\pi\varepsilon_0 R^2} = \frac{q}{4\pi\varepsilon_0 R^2}$$

$$F_{C-B} = \frac{(2q/3)(q/3)}{4\pi\varepsilon_0 \left(3R^2\right)} = \frac{q^2}{54\pi\varepsilon_0 R^2}$$

31. A radioactive sample S1 having an activity of $5\mu Ci$ has twice the number of nuclei as another sample S2 which has an activity of $10\mu Ci$. The half lives of S1 and S2 can be

(A) 20 years and 5 years, respectively
(B) 20 years and 10 years, respectively
(C) 10 years each
(D) 5 years each

Sol. (A)

$5\mu Ci = \lambda(2N)$

and $10\mu Ci = \lambda(N)$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{5}{20} = \frac{1}{4}$

$\Rightarrow \frac{T_1}{T_2} = \frac{4}{1}$

SECTION – II

Reasoning Type

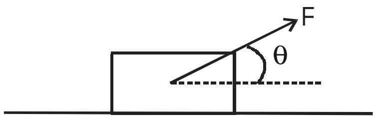
This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct

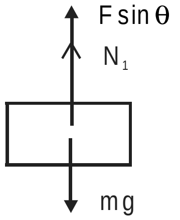
32. STATEMENT -1
It is easier to pull a heavy object than to push it on a level ground.
and
STATEMENT -1
The magnitude of frictional force depends on the nature of the surfaces in contact.

(A) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is a correct explanation for STATEMENT -1
(B) STATEMENT -1 is Trure, STATEMENT -2 is True; STATEMENT -2 **is NOT** a correct explanation for STATEMENT -1
(C) STATEMENT -1 is True, STATEMENT -2 is False
(D) STATEMENT -1 is False, STATEMENT -2 is True

Sol.

In pulling case

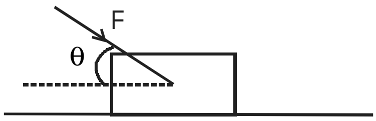


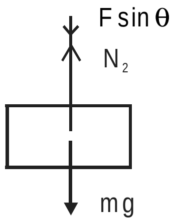


$\therefore \; N + F\sin\theta = mg$

$N_1 = (mg - F\sin\theta)$

In Pushing case





$N_2 = (F\sin\theta + mg)$

Normal force N_2 in pushing is more than the normal force N_1 in pulling.

$\therefore \; f = \mu N$

so if is easier to pull a heavy object than to push

so, statement - I is right & and of friction force also repel on surface of nature.

Therefore statement (I) & statement (II) both are right & statement (II) is not the correct explanation of statement (I)

A CAREER LAUNCHER PROGRAM

33. STATEMENT-1

For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

and

STATEMENT-2

The electrical potential of a sphere of radius R with charges Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol.

Both the statements are true, & statement (II) is not the correct explanation of statement (I). Because, whatever be the value of potential we can assign it zero as a reference.

34. STATEMENT-1

The sensitivity of a moving-coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

and

STATEMENT-2

Soft iron has a high magnetic permeability and cannot be easily mangetized or demagnetized.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol.

$$\text{Sensitivity} = \left(\frac{\text{range of output}}{\text{range of Input}}\right)$$

By placing suitable magnetic material as a were inside the coil, the magnetic moment of the coil will increase so, torque $\vec{\tau}$ which is equal to $MB\sin\theta$ will increase.

That's why we will find more range of output for a given range of input in a galvanometer.

$\therefore$ Statement (I) is correct

Highly permeable magnetic material easily magnetized & easily demagnetise.

$\therefore$ Statement (II) is wrong.

35. STATEMENT-1

For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

and

STATEMENT-2

If the observer and the object are moving at velocities $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\overrightarrow{V_2} - \overrightarrow{V_1}$.

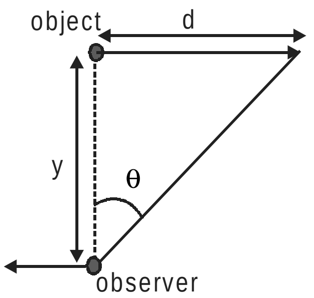
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol.



y = Distance in between observer & object

d = relative displacement between observer & object

$$\tan\theta = \left[\frac{d}{y}\right]$$

If y is very large , then angle subtended by displacement d in a given time is very small as compare to nearer object.

so statement (I) is correct.

Now, observer velocity w,r.t laboratory frame = V_1

& object " " " " = V_2

$\therefore$ Velocity of the object w.r.t observer $\vec{V}_{21} = \vec{V}_2 - \vec{V}_1$

so statement (II) is also correct.

SECTION –III

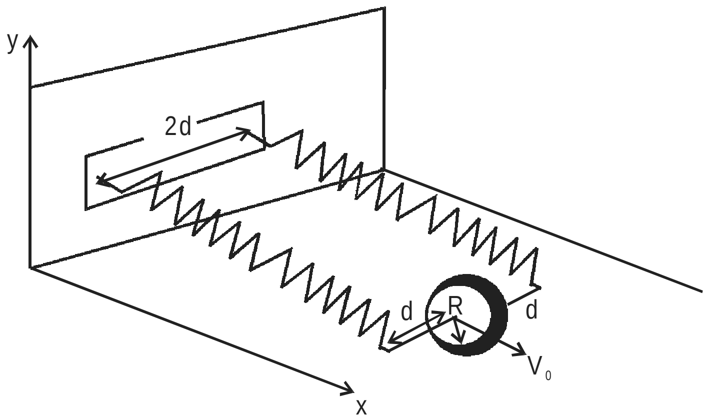
Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 36 to 38

A uniform thin cylindrical disk of mass M and radius R is attached to two id enthical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to hte axle of hte disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\overrightarrow{V_0} = V_0\hat{i}.$ The coefficient of friction is μ.

Figure :

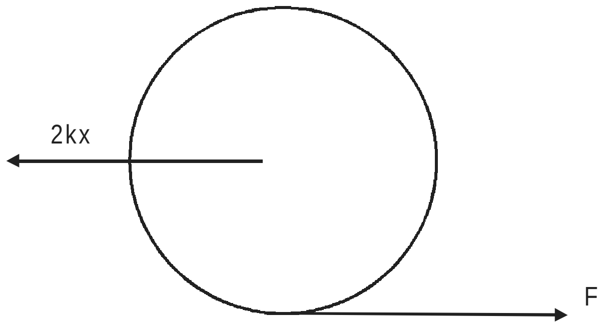


36. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) kx (B) –2kx (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

Sol.

for translational motion



2kx − F = Ma ....(1)
for rotational motion

$$FR = I\alpha = \frac{MR^2}{2}\left(\frac{a}{R}\right)\text{.............(2)}$$

from (1) and (2) $F = \frac{ma}{2}$

$$a = -\frac{4kx}{3m}$$

$$\text{Force} = Ma = -\frac{4kx}{3m}\ \text{(D)}$$

37. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

Sol. (D)

From the above question it is evident that net restoring force

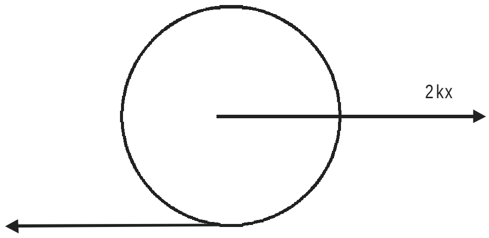
$$F = -\frac{4kx}{3M}$$

$$\therefore \omega = \sqrt{\frac{4k}{M}}$$

38. The maximum value of V_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

Sol. (C)



$2kx - f = Ma_{CM}$

$$fR = \frac{MR^2}{2}\left(\frac{a_{CM}}{R}\right)$$

$$f = \frac{Ma_{CM}}{2}$$

for slipping to start f should have its maximum value i.e, static friction

$$f = \frac{Ma_{CM}}{2} = \mu Mg$$

$$f = 2\mu gM \Rightarrow a_{CM} = 2\mu g$$

we know $a_{CM} = \frac{4kx}{3m}$

$$\therefore \frac{4kx}{3m} = 2\mu g$$

$$x = \frac{6\mu Mg}{4k}$$

hence at this x slipping will start the velocity required to attain this displacement is

$$2\times\frac{1}{2}kx^2 = \frac{1}{2}m\nu^2 + \frac{1}{2}I\omega^2$$

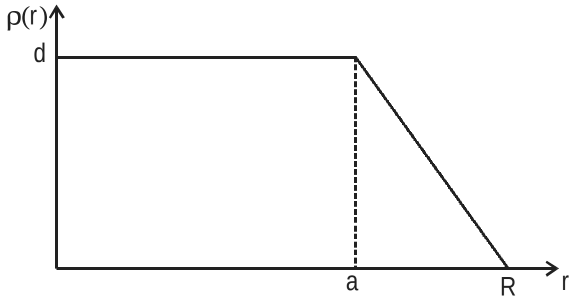
$$K\left(\frac{6\mu Mg}{4k}\right)^2 = mv^2 + \frac{MR^2}{2}\frac{\nu^2}{R^2}$$

$$\nu = \mu g\sqrt{\frac{3M}{k}}$$

Paragraph for Question Nos. 39 to 41

The nuclear charges (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r form the centre of the nucleus as shown in figure. The electric field is only along the radial direction.

Figure :



39. The electric field at r =R is

(A) independent of a
(B) directly proportional to a
(B) directly proportional to a^2
(D) inversely proportional to a

Sol. (A)

At r = R,

From Guass law,

$$E.4\pi R^2 = \frac{qen}{t_0} = \frac{Z_e}{t_0}$$

$$\Rightarrow E = \frac{Ze}{4\pi t_0 R^2}$$

E is independent of a.

40. For a = 0, the value of d (maximum value of ρ as shown in the figure) is

(A) $\frac{3Ze}{4\pi R^3}$
(B) $\frac{3Ze}{\pi R^3}$
(C) $\frac{4Ze}{3\pi R^3}$
(D) $\frac{Ze}{3\pi R^3}$

Sol. (B)

$$\rho(r) = -\frac{dr}{R} + d$$

$\therefore$ The charge inside the nucleus,

$$Ze = \int_0^R \rho(r).4\pi r^2 dr$$

$$\Rightarrow Ze = \int_0^R \left\{\left(-\frac{d}{R}.r + d\right)4\pi r^2\right\}dr$$

$$= 4\pi\left[-\frac{d}{R}.\frac{r^4}{4} + d.\frac{r^3}{3}\right]_0^R$$

$$= 4\pi d\left[-\frac{R^3}{4} + \frac{R^3}{3}\right]$$

$$\text{or, } Ze = \frac{\pi d R^3}{3}$$

$$\Rightarrow d = \frac{3Ze}{\pi R^3}$$

41. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

(A) $a = 0$ (B) $a = \frac{R}{2}$ (C) $a = R$ (D) $a = \frac{2R}{3}$

Sol. (C)

Electric field within the nucleus is linearly dependent on r is possible when the charge distribution is uniform.

$\therefore$ a = R

$$E.4\pi r^2 = \frac{qen}{t_0} = \frac{d.\frac{4}{3}\pi r^2}{t_0}$$

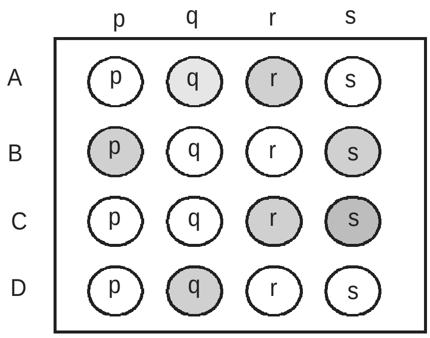
$$\Rightarrow E = \left(\frac{1}{3t_0}d\right)r$$

$E \propto r$

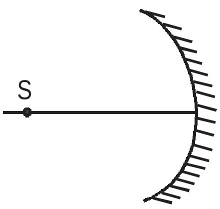
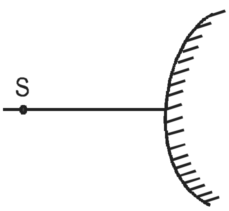
SECTION - IV

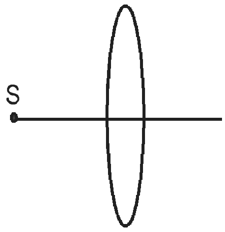
Matrix Match Type

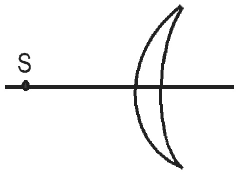
The section contains 3 questions. Each question contains statements given in two columns, which have to be matched. Statements in **Column I** are labelled as A, B, C and D whereas statements in **Columns II** are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example.
If the correct matches are A-q, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following:



42. An optical component and an object S placed along its optic axis are given in **Column I.** The distance between the object and the component can be varied. The properties of images are given in **Column II.** Match all the properties of images from **Column II .** Match all the properties of images from **Column II** with the appropriate components given in **Column I.** Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) 	(p) Real image
(B) 	(q) Virtual image

(C)  (r) Magnified image

(D)  (s) Image at infinity

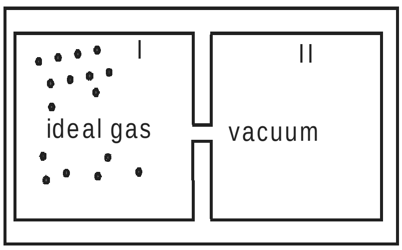
Sol.

(A) PQRS
(B) Q
(C) PQRS
(D) PQRS

43. **Column I** contains a list of processes involving expansion of an ideal gas. Match this with **Column II** describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I | **Column II**

(A) An insulated container has two chambers separated by a value. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened. — (p) The temperature of the gas decreases



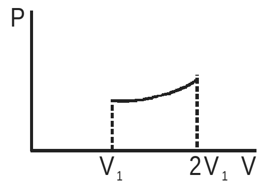
(B) An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas — (q) The temperature of the gas increases or remains constant

(C) An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume

(r) The gas loses heat

(D) An ideal monatomic gas expands such that its pressure P and volume V follows the behavior shown in the graph

(s) The gas gains heat



Sol. $A \to q$

$B \to p, r$

$C \to p, s$

$D \to q, s$

(A) Free expansion under adiabatic conditions

$\Rightarrow \Delta Q = 0 \;\; \& \; \Delta Q = 0$

Hence $\Delta U = 0$

$\Rightarrow$ T remains constant

(B) $P \propto \frac{1}{V^2}$ or $P = \frac{k}{V^2}$

PV =nRT

or $\frac{K}{V} = nRT$

$\Rightarrow V \to 2V$

$T \to \frac{T}{2}$

$\Rightarrow \Delta U = \frac{3}{2} nR\Delta T = -\frac{3K}{4V}$

$\Delta W = \int_{V}^{2V} PdV = R\int_{V}^{2V} \frac{dV}{V^2} = \frac{K}{2V}$

$\Rightarrow \Delta Q = \Delta U + \Delta W < 0$

(C) $P \propto \frac{1}{V^{4/3}}$

$$P = \frac{K}{V^{4/3}}$$

$$\Rightarrow \frac{K}{V^{1/3}} = nRT$$

$$V \to 2V$$

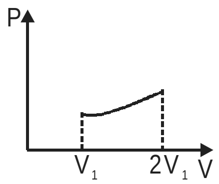
$$T \to \frac{T}{2^{1/3}}$$

$$\Delta U = \frac{3}{2} nR\Delta T = \frac{3K}{2V^{1/3}}\left[\frac{1}{2^{1/3}} - 1\right]$$

$$\Delta W = \int_{V}^{2V} \frac{KdV}{V^{4/3}} = \frac{3K}{V^{1/3}}\left[1 - \frac{1}{2^{1/3}}\right]$$

$$\Rightarrow \Delta Q = \Delta U + \Delta W > 0$$

(D)



$$PV = nRT$$

$$2P'V = nRT'$$

$$\Rightarrow \frac{T'}{T}\left(\frac{2P'}{P}\right) > 1$$

$\Delta W > 0$ (area under curve)

$$\Delta U = \frac{3}{2} nR\Delta T = \frac{3}{2} V\left[2P' - P\right] > 0$$

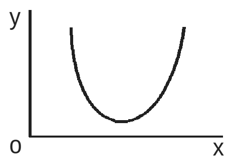
$$\Rightarrow \Delta Q = \Delta U + \Delta W > 0$$

44. **Column I** give a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II.** Match the set of parameters given in **Column I** with the graph given in **Column II.** Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I | **Column II**

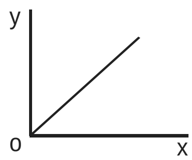
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)

(p)



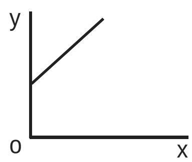
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction

(q)



(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle

(r)



(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)

(s)

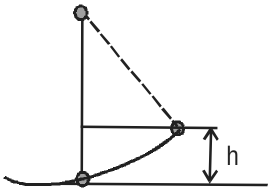
Sol. $A \to q, r$

$B \to q, s$

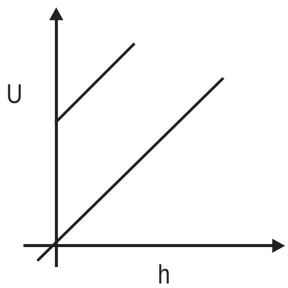
$C \to s$

$D \to q$

(A) U = mgh (reference at lowest point)



$U - U_0 = mgh$ (reference at any point)



(B) (i) a > 0, v > 0

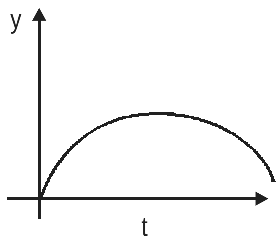
(ii) a < 0, v > 0

(iii) a = 0 , v > 0

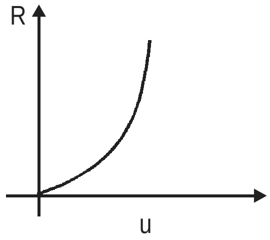
$(iii) \to q, r \qquad (y = y_0 + vt)$

$(i) \to s \qquad \left(y = y_0 + vt + \frac{1}{2}at^2\right)$

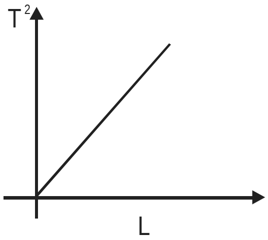
(ii)



(C) $R \propto u^2$



(D) $T^2 \propto L$



CHEMISTRY

PART III

Section – I

Straight Objective Type

This section contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

45. Solubility product constants (K_{sp}) of salts of types MX, MX_2 and M_3X at temperature "T" are 4.0×10^{-8}, 3.2×10^{-14} and 2.7×10^{-15}, respectively. Solubilities (mol dm^{-3}) of the salts at temperature "T" are in the order

(A) $MX > MX_2 > M_3X$ (B) $M_3X > MX_2 > MX$ (C) $MX_2 > M_3X > MX$ (D) $MX > M_3X > MX_2$

Sol. (D)

Solubility of MX = 0.0002 moles/dm^3
Solubility of MX_2 = 0.00002 moles/dm^3
Solubility of MX_3 = 0.0001

46. Electrolysis of dilute aqueous NaCl solution was carried out by passing 10 milli ampere current. The time required to liberate 0.01 mol of H_2 gas at the cathode is (1 Faraday = 96,500 C mol^{-1})

(A) 9.6×10^4 sec (B) 19.3×10^4 sec (C) 28.95×10^4 sec (D) 38.6×10^4 sec

Sol. (B)

$w = zIt$ or $n \rightarrow$ number of moles = $\dfrac{It}{96500\ n \rightarrow n-\text{factor}}$

$$0.01 = \frac{10\times10^{-3}\times f}{96{,}500\times2}$$

$$t = 19.3\times10^4 \text{ sec}.$$

47. Among the following, the surfactant that will form micelles in aqueous solution at the lowest molar concentration at ambient conditions is

(A) $CH_3(CH_2)_{15}N^+(CH_3)_3Br^-$ (B) $CH_3(CH_2)_{11}OSO_3^-Na^+$

(C) $CH_3(CH_2)_6COO^-Na^+$ (D) $CH_3(CH_2)_{11}Na^+(CH_3)_3Br^-$

Sol. (B)

48. Both $\left[Ni(CO)_4\right]$ and $\left[Ni(CN)_4\right]^{2-}$ are diamagnetic. The hybridisations of nickel in these complexes, respectively are

(A) sp^3, sp^3 (B) sp^3, dsp^2 (C) dsp^2, sp^3 (D) dsp^2, dsp^2

Sol. (B)

$Ni(CO)_4 \longrightarrow sp^3$

$\left[Ni(CN)_4\right]^2 \longrightarrow dsp^2$

49. The IUPAC name of $\left[Ni(NH_3)_4\right]\left[NiCl_4\right]$ is

(A) Tetrachloronickel (II) - tetraamminenickel (II)
(B) Tetraamminenickel (II) - tetrachloronickel (II)
(C) Tetraamminenickel (II) - tetrachloronickelate (II)
(D) Tetrachloronickel (II) - tetraamminenickelate (0)

Sol. (C)

Tetrammine nickel (II) - tetrachloro nickelate (II)

50. Among the following, the coloured compound is

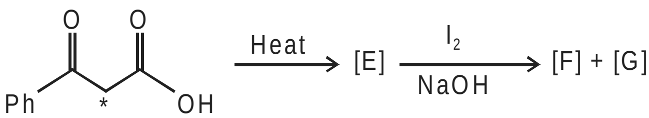
(A) CuCl (B) $K_3\left[Cu(CN)_4\right]$

(C) CuF_2 (D) $\left[Cu(CH_3CN)_4\right]BF_4$

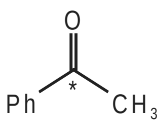
Sol. (C)

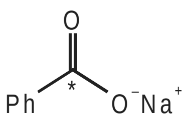
CuF_2

51. In the following reaction sequence, the correct structures of E, F and G are

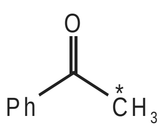


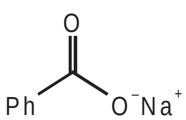
(* implies ^{13}C labelled carbon)

(A) E = 

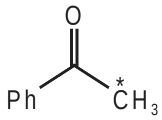
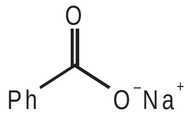
F = 

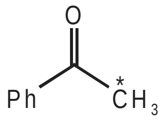
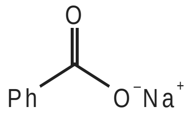
G = CHI_3

(B) E = 

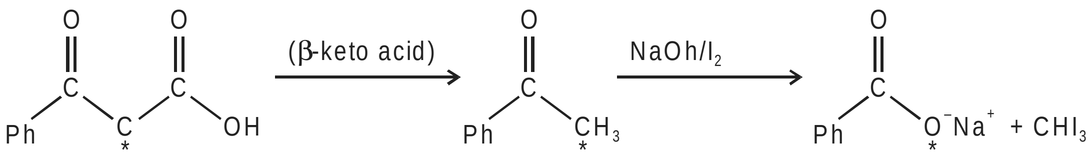
F = 

G = CHI_3

(C) E =  F =  G = $\overset{*}{C}HI_3$

(D) E =  F =  G = $\overset{*}{C}H_3I$

Sol. (C)



52. The correct stability order for the following species is

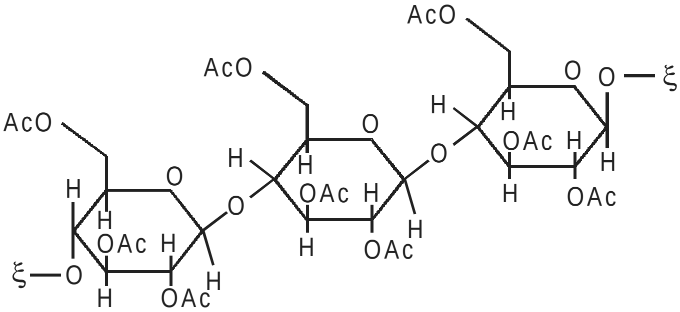
I II III IV

(A) I > IV > I > III (B) I > II > III > IV (C) II > I > IV > III (D) I > III > II > IV

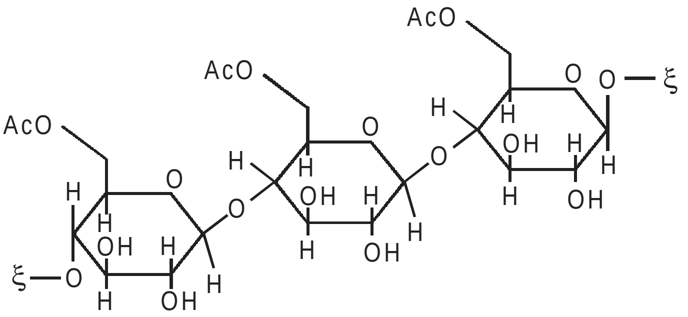
Sol. (D)

I is tertiary and resonance stabilised.
III is secondary and resonance stabilised.
II is secondary only
IV is primary only.

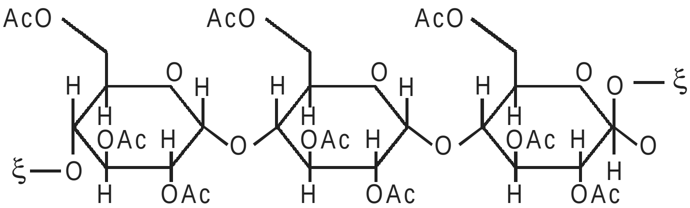
53. Cellulose upon acetylation with excess acetic anhydride/H_2SO_4 (catalytic) gives cellulose triacetate whose structure is

(A) 

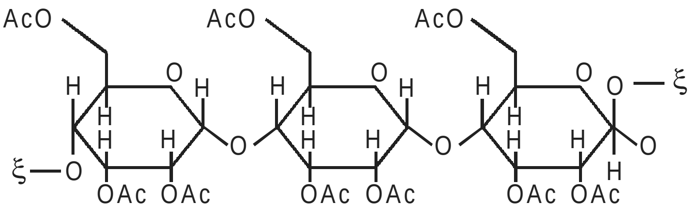
(B)



(C)



(D)



Sol. (A)

Cellulose is made up from β-D glucose.

Section II

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

54. STATEMENT-1: $\left[Fe(H_2O)_5 NO\right]SO_4$ is paramagnetic.

and

STATEMENT-2: The Fe in $\left[Fe(H_2O)_5 NO\right]SO_4$ has three unpaired electrons.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

Sol. (A)

Oxidation state of Fe is +1, charges on NO is +1 and Fe^+ has three unpaired electrons.

55. STATEMENT-1: The geometrical isomers of the complex $\left[M(NH_3)_4 Cl_2\right]$ are optically inactive.

and

STATEMENT-2: Both geometrical isomers of the complex $\left[M(NH_3)_4 Cl_2\right]$ possess axis of symmetry.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

Sol. (A)

56. STATEMENT-1: There is a natural asymmetry between converting work to heat and converting heat to work

and

STATEMENT-2: No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

Sol. (A)

57. STATEMENT-1 : Aniline on reaction with $NaNO_2$/HCl at 0 °C followed by coupling with β-naphthol gives a dark blue coloured precipitate.

and

STATEMENT-2 : The colour of the compound formed in the reaction of aniline with $NaNO_2$/HCl at 0 °C followed by coupling with β-naphthol is due to the extended conjugation.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

Sol. (D)

SECTION - III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 58 to 60

In hexagonal systems of crystals, a frequently encountered arrangement of atoms is described as a hexagonal prism. Here, the top and bottom of the cell are regular hexagonas and three atoms are sandwiched in between them. A space-fillingt model of this structure, called hexagonal close-packed (HCP), is contituted of a sphere on a flat surface surrounded in the same plane by six identical spheres as closely as possible. Three spheres are then placed over the first layer so that they touch each other and represent the second layer. Each one of these three spheres touches three spheres of the bottom layer. Finally, the second layer is covered with a third layer that is identical to the bottom layer in relative position. Assume radius of every sphere to be ‘r’.

58. The number of atoms in this HCP unit cell is

(A) 4 (B) 6 (C) 12 (D) 17

Sol. (B)

6

59. The volume of this CHP unit cell is

(A) $24\sqrt{2}r^3$ (B) $16\sqrt{2}r^3$ (C) $12\sqrt{2}r^3$ (D) $\frac{64}{3\sqrt{3}}r^3$

Sol. (A)

$24\sqrt{2}r^3$

60. The empty space in this HCP unit cells is

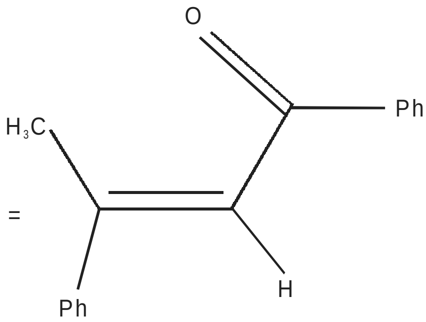
(A) 74% (B) 47.6% (C) 32% (D) 26%

Sol. (D)

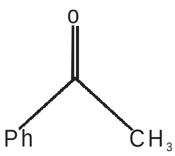
Packing fraction is HCP unit cell is 74% $\therefore$ the emptyspace is 100 – 74 = 26%

Paragraph for Question Nos. 61 to 63

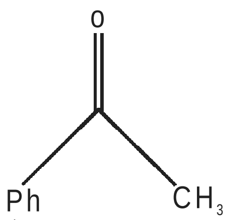
A t tertiary alcohol **H** upon acid catalysed dehydration gives a product **I**. Ozonolysis of **I** leads to compounds **J** and **K**. Compound **J** upon reaction with KOH gives benzyl alcohol and a compound **L,** whereas **K** on reaction with KOH gives only M.

M =


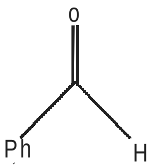
61. Compound **H** is formed by the rection of

(A) 

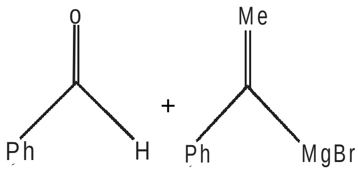
+ PhMgBr

(B) 

+ $PhCH_2MgBr$

(C) 

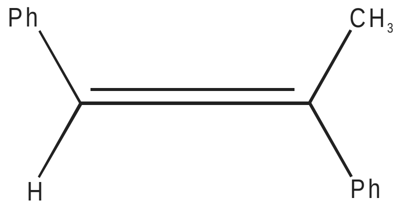
+ $PhCH_2MgBr$

(D) 

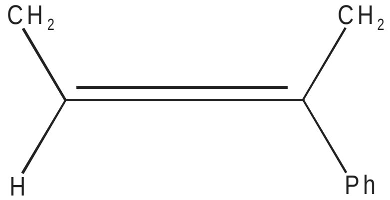
Sol. (B)

62. The structure of compound **I** is

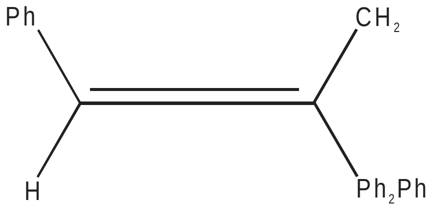
(A)



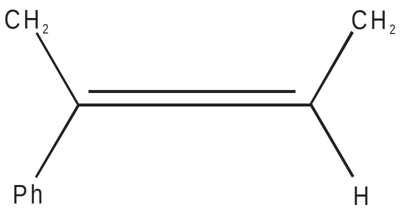
(B)



(C)



(D)

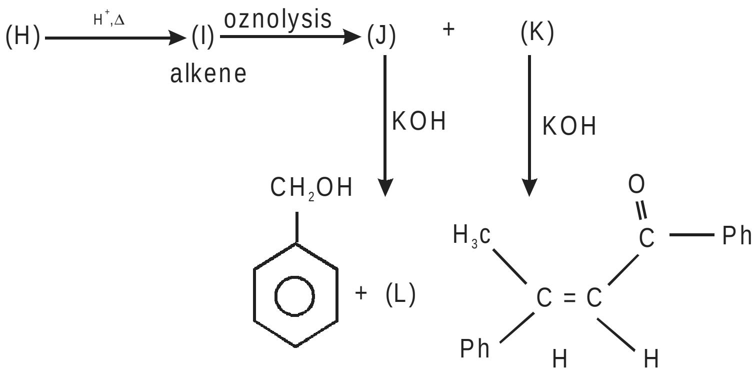


Sol. (D)

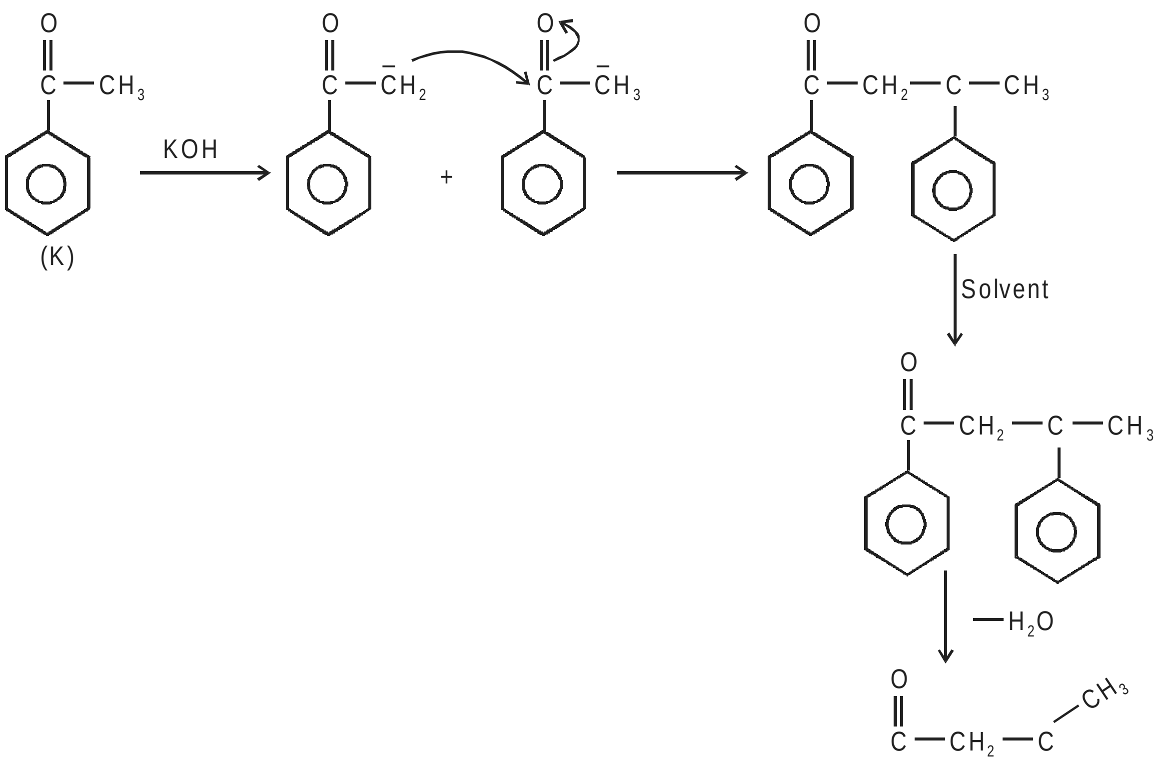
63. The structures of compounds **J, K** and **L,** respectively, are
(A) $PhCOCH_3$, $PhCH_2COCH_3$ and $PhCH_2COO^-K^+$
(B) $PhCHO$, $PhCH_2CHO$ and $PhCOO^-k^+$
(C) $PhCOCH_3$,$PhCH_2CHO$ and $CH_3COO^-K^+$
(D) $PhCHO$, $PhCOCH_3$ and $PhCOO^-K^+$

Sol. (D)

Explanation for questions 61 to 63



Reaction $k \xrightarrow{KOH} M$ is aldol condensation compound K is acetophenone



SECTION - IV

Maxtrix Match Type

This section contains 3 questions. Each question contains statements given in two columns, which have to be matched. Statements in **Column I** are labelled as A, B, C and D whereas statements in **Column II** are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example.
If the correct matches are A-q, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following.

64. Match the conversions in **Column I** with the type(s) of reaction(s) given in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS

Column I	**Column II**
(A) $PbS \rightarrow PbO$	(p) roasting
(B) $CaCO_3 \rightarrow CaO$	(q) Calcination
(C) $ZnS \rightarrow Zn$	(r) carbon reduction
(D) $Cu_2S \rightarrow Cu$	(s) self reduction

Sol.

A - p
B - q
C - p, r
D - p, s

65. Match the entries in **Column I** with the correctly related quantum number(s) in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS

Column I	**Column II**
(A) Orbital angular momentum of the electronin a hydrogen-like atomic orbitalb	(p) Principal quantum number
(B) A hydrogen-like one-electron wave function obeying Pauli principle	(q) Azimuthal quantum number
(C) Shape, size and orientation of hydrogen -like atomic orbitals	(r) Magnetic quantum number
(D) Probability density of electron at the nucleus in hydrogen-like atom	(s) Electron spin quantum number

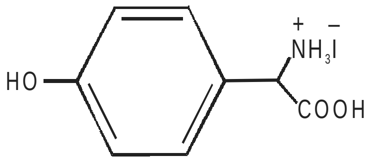
Sol.

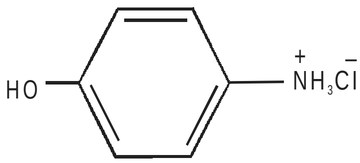
A - q
B - s
C - p, q, r
D - p

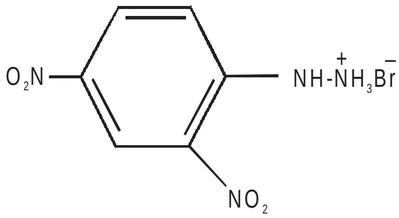
66. Match the compounds in **Column I** with their characteristic test(s)/reactions(s) given in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I

(A) $H_2N-\overset{\oplus}{N}H_3\ \overset{\ominus}{Cl}$

(B) 

(C) 

(D) 

Column II

(p) sodium fusion extract of the compound give Prussian blue colour with $FeSO_4$

(q) gives positive $FeCl_3$ test

(r) gives white precipitate with $AgNO_3$

(s) reacts with aldehydes to form the corresponding hydrazone derivative

Sol.

A - r, s
B - q, p
C - r, q, p
D - p