Code 2

MATHEMATICS

PAPER - I

SECTION – I
Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

For the benefit of 11th/12th Studying students, we have (*) marked the questions which are from 11th syllabus. You are advised to solve these questions in 100 minutes.

1. If $0 < x < 1$, then $\sqrt{1+x^2}\ [\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2} =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

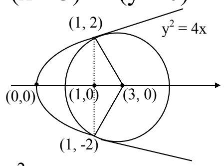
Sol $\sqrt{1+x^2}\left[\left\{\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right\}^2 - 1\right]^{1/2}$

$= \sqrt{1+x^2}\ \left[1+x^2-1\right]^{1/2} = x\ \sqrt{1+x^2}$.

Key (C)

*2. Consider the two curves
$C_1 : y^2 = 4x$
$C_2 : x^2 + y^2 - 6x + 1 = 0$
Then,
(A) C_1 and C_2 touch each other only at one point
(B) C_1 and C_2 touch each other exactly at two points
(C) C_1 and C_2 intersect (but do not touch) at exactly two points
(D) C_1 and C_2 neither intersect nor touch each other

Sol $(x-3)^2 + (y-0)^2 = (2\sqrt{2})^2$



$x^2 + 4x - 6x + \ = 0$
$(x-1)^2 = 0$
$x = 1, 1$
Ans. touch each other exactly two points.

Key (B)

3. The edges of a parallelopiped are of unit length and are parallel to non–coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = \frac{1}{2}$

Then, the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol $V^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}$

$\Rightarrow V^2 = \frac{1}{2}$

$\Rightarrow V = \frac{1}{\sqrt{2}}$

Key (A)

*4. Let a and b be non–zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

Sol $x^2 - 5xy + 6y^2 = 0$ represent a pair of lines passing through origin

$ax^2 + ay^2 = -c$

$\Rightarrow x^2 + y^2 = \frac{-c}{a} > 0$

$\Rightarrow ax^2 + ay^2 + c = 0$ represent a circle

Key (B)

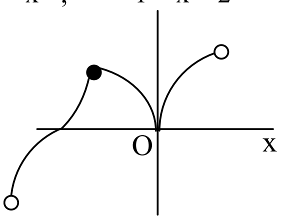
5. The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is

(A) 0 (B) 1

(C) 2 (D) 3

Sol $f(x) = (2 + x)^3, -3 < x \le -1$

$= x^{2/3}, \quad -1 < x < 2$



The total number of local maximum or minimum = 2.

Key (C)

6. Let $g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}$; $0 < x < 2$, m and n are integers, $m \ne 0$, $n > 0$, and let p be the left hand derivative of $|x - 1|$ at x = 1. If $\lim_{x \to 1+} g(x) = p$, then

(A) n = 1, m = 1 (B) n = 1, m = –1

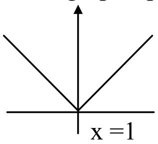
(C) n = 2, m = 2 (D) n > 2, m = n

Sol According to question we get,

$$\lim_{x\to 1^+} g(x) = -1 \Rightarrow \lim_{h\to 0^+} \frac{(h)}{\log\cos^2(h)} = \lim_{h\to 0^+} \frac{(h)^2}{2\log(\cosh)} = \lim_{h\to 0} \frac{-2h}{2\tanh} = -1$$

Alternate:

From graph : p = –1



$$g(x) = \frac{(x-1)^n}{\log(\cos^m(x-1)} \quad 0 < x < 2,\ m, \neq 0 \quad n \in N$$

$$g(x) = \lim_{x\to 1^+} \frac{(x-1)^n}{\log\cos^m(x-1)} = -1$$

$$g(1+) = \lim_{h\to 0} \frac{(h)^n}{\log\cos^m h},\ h > 0 = \lim_{h\to 0} \frac{h^n}{m(\ln\cosh)} = \frac{1}{m}\lim_{h\to 0} \frac{h^n}{(\ln\cosh)}$$

$$= \frac{1}{m}\lim_{h\to 0} \frac{n\,h^{n-1}}{-\sinh} \times \cosh = \frac{-n}{m}\lim_{h\to 0} \frac{h^{n-2}}{\left(\frac{\sinh}{h}\right)} \times (\cosh) \Rightarrow n = 2,\ m = 2$$

Key (C)

SECTION – II

Multiple Correct Answer Type

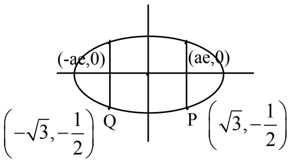
This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

*7. Let P (x_1, y_1) and Q(x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2 + 2\sqrt{3}\ y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}\ y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}\ y = 3 - \sqrt{3}$ (D) $x^2 - 2\sqrt{3}\ y = 3 - \sqrt{3}$

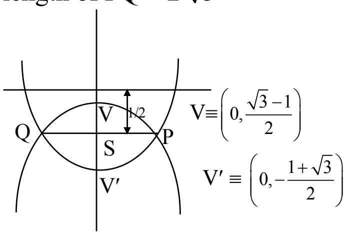
Sol Given ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1 \Rightarrow e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$



$P \equiv (ae, -b^2/a) = \left(\sqrt{3}, -\frac{1}{2}\right)$

$Q \equiv (-ae, -b^2/a) = (-\sqrt{3}, -\frac{1}{2})$

length of PQ = $2\sqrt{3}$



$VS = SV' = \frac{PQ}{4} = \frac{\sqrt{3}}{2}$

$\therefore$ Equations of parabolas are

$x^2 = -2\sqrt{3}\left(y - \frac{\sqrt{3}-1}{2}\right) \Rightarrow x^2 + 2\sqrt{3}\ y = 3 - \sqrt{3}$

and $x^2 = 2\sqrt{3}\left(y + \frac{1+\sqrt{3}}{2}\right) \Rightarrow x^2 - 2\sqrt{3}\ y = 3 + \sqrt{3}$

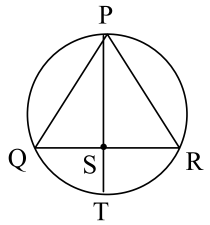
Key (B), (C)

*8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (B) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (D) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

Sol



H.M. < G.M.

$\frac{2}{\frac{1}{PS} + \frac{1}{ST}} < (PS.\ ST)^{1/2} \Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{PS.ST}}$

As PS × ST = QS × SR

$\Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$ ... (i) Option (B) is correct

A.M. > G.M

$\frac{QS + SR}{2} > \sqrt{(QS.SR)}\ \therefore\ \sqrt{QS \times SR} < \frac{QR}{2}$ ... (ii)

From (i) and (ii)

$\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

Key (D)

9. Let f(x) be a non–constant twice differentiable function defined on $(-\infty, \infty)$ such that f(x) = f(1–x) and $f'\left(\frac{1}{4}\right) = 0$. Then,

(A) $f''(x)$ vanishes at least twice on [0, 1] (B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x\, dx = 0$ (D) $\int_{0}^{1/2} f(t) e^{\sin \pi t}\, dt = \int_{1/2}^{1} f(1-t)\ e^{\sin \pi t}\ dt$

Sol (A) $f'(x) = -f'(1 - x)$

$f'(1/4) = -f'(3/4) = 0$

$f'(3/4) = 0$
$f'(1/4) = 0$
$\Rightarrow \exists$ a point $\in (1/4, 3/4)$ in which $f''(x) = 0$
$f''(1/2) = 0$

(B) $f(x) = f(1-x)$
$f'(x) = -f'(1-x)$
$f'(1/2) = -f'(1/2)$
$f'(1/2) = 0.$
(C) $f(x + 1/2) = f(1 - x - 1/2)$
$\Rightarrow f(x + 1/2) = f(1/2 - x)$
i.e., $f(x + 1/2)$ is even
i.e., $\int_{-1/2}^{1/2} f(x+1/2)\sin x = 0$
(D) $\int_{0}^{1/2} f(t)\, e^{\sin\pi t} = \int_{1/2}^{1} f(1-t)\ e^{\sin\pi t}\, dt$
Put $(1 - t) = u$
$\Rightarrow$ R.H.S. $= -\int_{1/2}^{0} f(4)\ e^{\sin\pi(1-u)}\, du = \int_{0}^{1/2} f(u)\ e^{\sin\pi u}\, dx$

Key (A), (B), (C), (D)

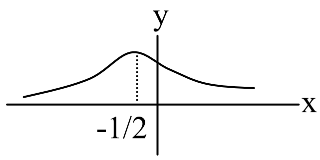
10. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for n = 1, 2, 3, ... then ,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

Sol Consider: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{k^2 + kn + n^2} = \int_0^1 \frac{dx}{x^2 + x + 1} = \frac{\pi}{3\sqrt{3}}$

$f(x) = \frac{1}{x^2 + x + 1} = \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$



As f(x) is decreasing for x > 0, $S_n < \int_0^1 \frac{1}{x^2 + x + 1} dx < T_n$

Hence A & D are correct.

Key (A), (D)

SECTION – III
Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

11. Consider the system of equations
$x - 2y + 3z = -1$
$-x + y - 2z = k$
$x - 3y + 4z = 1$

STATEMENT–1: The system of equations has no solutions for $k \neq 3$
and
STATEMENT–2: The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol.: $\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$

$\Delta_z = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & k \\ 1 & -3 & 1 \end{vmatrix} = 3 - k \neq 0, k \neq 3$

Key (A)

12. Consider the system of equations ax + by = 0, cx + dy = 0, where a, b, c, d$\in$ {0, 1}
STATEMENT–1: The probability that the system of equations has a unique solution is 3/8
and
STATEMENT–2: The probability that the system of equations has a solution is 1.

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol a, b, c, d $\in$ {0, 1}
$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $n(s) = 2^4 = 16$
(I) To have unique solution $\Delta \neq 0$.
$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0, & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$ Total = 6 cases
The probability that system of equations has unique solution = 6/16 = 3/8
II. Homogenous system is always consistent

Key (B)

13. Let f and g be real valued functions defined on interval (–1, 1) such that $g''(x)$ is continuous $g(0) \neq 0$, $g'(0) = 0$ $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.
STATEMENT–1: $\lim_{x \to 0}$ [g(x) cotx – g(x) cosec x] = $f''(0)$.
and
STATEMENT–2: $f'(0) = g(0)$
(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1

(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol $f(x) = g(x) \sin x$
$f''(x) = g''(x) \sin x + 2g'(x) \cos x + g(x) \sin x$
$f''(0) = 2g'(0) = 0$
$f'(x) = g'(x) \sin x + g(x) \cos x$
$f'(0) = g(0)$
For statement 1: $\lim_{x\to 0} \frac{g(x)\cos x - g(0)}{\sin x} = \lim_{x\to 0} \frac{g'(x)\cos x - g(x)\sin x}{\cos x} = g'(0) = 0 = f''(0)$
For statement 2
$f'(0) = g(0)$
Key (B)

14. Consider three planes $P_1 : x - y + z = 1$
$P_2 : x + y - z = -1$
$P_3 : x - 3y + 3z = 2$
Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively
STATEMENT–1: At least two of the lines L_1, L_2 and L_3 are non-parallel
and
STATEMENT–2: The three planes do not have a common point

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol L_1, L_2, L_3 are parallel to each other $\Rightarrow$ statement (1) is not true
$$D = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = 0$$
$\Rightarrow$ The three planes do not have a common point.
$\Rightarrow$ statement (2) is correct.
Key (D)

SECTION – IV
Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 15 to 17

Let A, B, C be three sets of complex numbers as defined below
$A = \{z : \text{Im} z \geq 1\}$
$B = \{z : |z - 2 - i| = 3\}$
$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$

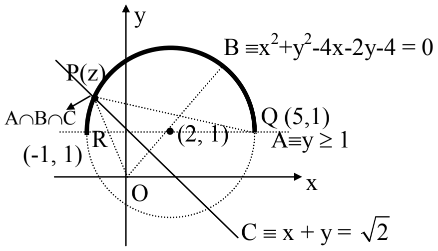
*15. The number of element in the set $A \cap B \cap C$ is
(A) 0 (B) 1
(C) 2 (D) ∞

*16. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between
(A) 25 and 29 (B) 30 and 34
(C) 35 and 39 (D) 40 and 44

*17. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(A) –6 and 3
(B) –3 and 6
(C) –6 and 6
(D) –3 and 9

Sol (15 – 17)



15. From graph only one point in $A \cap B \cap C$

Key (B)

16. $|z + 1 - i|^2 + |z - 5 - i|^2$
$= PR^2 + PQ^2 = RQ^2 = 6^2 = 36$

Key (C)

17. As $||z| - |\omega|| < |z - \omega| < 6$
$\Rightarrow ||z| - |\omega|| < 6$
$\Rightarrow -6 < |z| - |\omega| < 6$
$\Rightarrow -3 < |z| - |\omega| + 3 < 9$

Key (D)

Paragraph for Questions Nos. 18 to 20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D,E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

*18. The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$
(B) $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$
(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$
(D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

*19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$
(B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$
(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
(D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

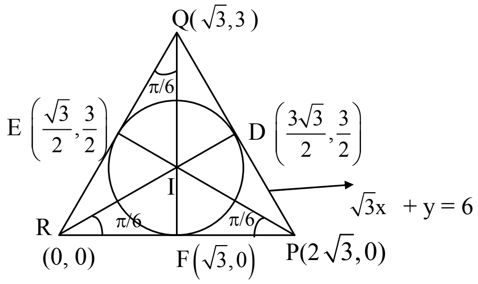
*20. Equations of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$
(B) $y = \frac{1}{\sqrt{3}}x, y = 0$
(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$
(D) $y = \sqrt{3}x, y = 0$

Sol 18–20

$$P \equiv \left(\frac{3\sqrt{3}}{2} - \sqrt{3} \times \left(-\frac{1}{2}\right), \frac{3}{2} - \sqrt{3} \times \frac{\sqrt{3}}{2}\right)$$

$= \left(2\sqrt{3}, 0\right)$



$I = (\sqrt{3}, 1)$

18. $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Key **(D)**

19. $E = \sqrt{3}\cos\frac{\pi}{3}, \sqrt{3}\sin\frac{\pi}{3} \equiv \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

$F \equiv (\sqrt{3}, 0)$

Key **(A)**

20. Equation of PR, y = 0

Equation of QR, $y = \sqrt{3}\,x$

Key **(D)**

Paragraph for Questions Nos. 21 to 23

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.

If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

21. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$

(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

22. The area of the region bounded by the curve y = f(x), the x–axis, and the lines x = a and x = b, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3[((f(x))^2 - 1]}dx + bf(b) - af(a)$ (B) $-\int_a^b \frac{x}{3((f(x))^2 - 1)}dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3((f(x))^2 - 1)}dx - bf(b) + af(a)$ (D) $-\int_a^b \frac{x}{3((f(x))^2 - 1)}dx - bf(b) + af(a)$

23. $\int_{-1}^{1} g'(x)\,dx =$

(A) $2g(-1)$ (B) 0

(C) $-2g(1)$ (D) $2g(1)$

Sol.: **21 – 23**

$y^3 - 3y + x = 0$

$\Rightarrow 3y^2 y' - 3y' + 1 = 0 \Rightarrow y' = \dfrac{1}{3(1-y^2)}$

$\Rightarrow y'' = \dfrac{6y(y')^2}{3(1-y^2)} = \dfrac{2y}{(1-y^2)^3.9} = \dfrac{2.2\sqrt{2}}{(1-8)^3.9} = -\dfrac{4\sqrt{2}}{7^3 3^2}$

Key **(B)**

22. $x = -y^3 + 3y$

as $x \in (-\infty, -2) \Rightarrow x < -2$

$\Rightarrow -y^3 + 3y < -2$

$\Rightarrow y^3 - 3y - 2 > 0 \Rightarrow (y+1)^2 (y-2) > 0$

$\Rightarrow y > 2\ \forall\ x \in (-\infty, -2)$ (As $y = -1 \Rightarrow x = -2$)

$\Rightarrow f(x)$ is positive $\forall x \in (-\infty, -2)$

Hence required area = $\int_a^b f(x)\,dx = \int_a^b y \cdot 1 dx = yx\big|_a^b - \int_a^b \frac{dy}{dx} x dx$

$= \int_a^b \dfrac{xdx}{3((f(x))^2 - 1)} + bf(b) - af(a)$

Key **(A)**

23. Consider: $(g(x))^3 - 3g(x) + x = 0$

and $(g(-x))^3 - 3g(-x) - x = 0$

$\Rightarrow (g(x))^3 + (g(-x))^3 - 3(g(x) + g(-x)) = 0$

$\Rightarrow [g(x) + g(-x)]\,[(g(x))^2 + (g(-x))^2 - g(x)\,g(-x) - 3] = 0$

Let $(g(x))^2 + (g(-x))^2 - g(x)\,g(-x) - 3 = 0$

$\Rightarrow g(0)^2 = 3 \Rightarrow g(0) = +\sqrt{3}$ or $-\sqrt{3}$ which is not the case as $g(0) = 0$, given

$\Rightarrow g(x) + g(-x) = 0 \Rightarrow g(x)$ is an odd function for $x \in (-2, 2)$

$\Rightarrow \int_{-1}^{1} g'(x)\,dx = g(1) - g(-1) = 2g(1)$.

Key **(D)**

PHYSICS

Useful Data:

Planck's constant $h = 4.1 \times 10^{-14}$ eV.s

Velocity of light $c = 3 \times 10^8$ m/s.

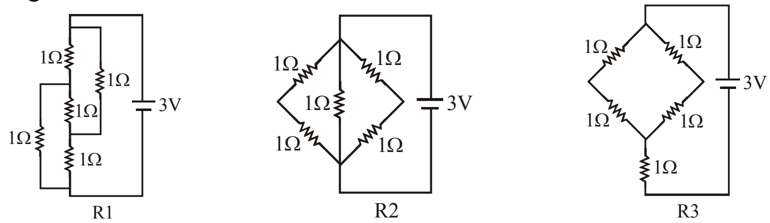
SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

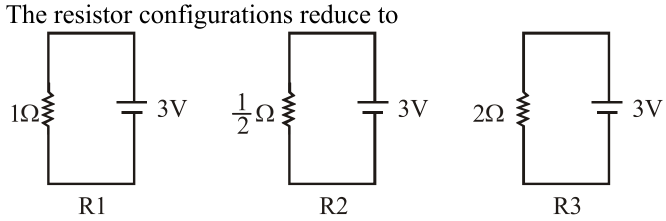
24. Figure shows three resistor configurations R1, R2 and R3 connected to 3V battery. If the power dissipated by the configuration R1, R2 and R3 is P1, P2 and P3, respectively, then

Figure :



(A) P1 > P2 > P3
(B) P1 > P3 > P2
(B) P2 > P1 > P3
(D) P3 > P2 > P1.

Sol. The resistor configurations reduce to



Now $P = \frac{V^2}{R_{eq}}$

$\therefore$ P2 > P1 > P3

Key **(C) is correct**.

25. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.
Least count for length = 0.1 cm
Least count for time = 0.1 s.

Student	**Length of the pendulum (cm)**	**Number of oscillations (n)**	**Total time for (n) oscillations (s)**	**Time period (s)**
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students I, II and III, respectively,

(A) $E_I = 0$
(B) E_I is minimum
(C) $E_I = E_{II}$
(D) E_{II} is maximum.

Sol. $g = 4\pi^2 \frac{\ell}{T^2}$, $E = \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$

Student	Length of the pendulum, error (cm), ℓ, $\Delta\ell$	No. of Oscillation	Time Period, error T, ΔT
I	64.0±0.1	8	$16.0 \pm \frac{0.1}{8}$
II	64.0±0.1	4	$16.0 \pm \frac{0.1}{4}$
III	20.0±0.1	4	$9.0 \pm \frac{0.1}{4}$

$E_I = \frac{0.2}{64}$, $E_{II} = \frac{0.3}{64}$, $E_{III} = \frac{0.1}{20} + \frac{0.1}{18}$.

Key E_1 is the minimum correct choice is (B)

26. Which one of the following statement is WRONG in the context of X–rays generated from X–ray tube ?
(A) wavelength of characteristic X–rays decreases when the atomic number of the target increases
(B) cut–off wavelength of the continuous X–rays depends on the atomic number of the target
(C) intensity of the characteristic X–rays depends on the electrical power given to the X–ray tube
(D) cut–off wavelength of the continuous X–rays depends on the energy of the electrons in the X–ray tube.

Sol. Cut–off wavelength depends upon accelerating potential.
Key (B) is the correct choice.

27. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60º). In the position of minimum deviation, the angle of refraction will be
(A) 30º for both the colours
(B) greater for the violet colour
(C) greater for the red colour
(D) equal but not 30º for both the colours.

Sol. In the position of minimum deviation

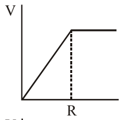
$r = \frac{A}{2}$ irrespective of colour

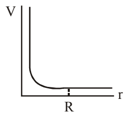
Key (A)

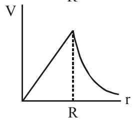
28. A spherically symmetric gravitational system of particles has a mass density

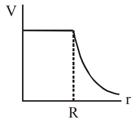
$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

Sol. For points $r \le R$,

$$\frac{G\left(\rho_0 \frac{4}{3}\pi r^3\right)m}{r^2} = \frac{mv^2}{r} \Rightarrow v \propto r$$

for points $r > R$

$$\frac{G\left(\rho_0 \frac{4}{3}\pi R^3\right)m}{r^2} = \frac{mv^2}{r} \Rightarrow v \propto \frac{1}{\sqrt{r}}$$

Key **(C)**

29. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$

(C) $\frac{3}{T}$ (D) $\frac{4}{T}$.

Sol Coefficient of volume expansion $\gamma = \frac{1}{V}\frac{dV}{dT}$

Now, $PT^2 = \text{constant} \Rightarrow \frac{T^3}{V} = \text{constant} \therefore \frac{3T^2.V - T^3\frac{dV}{dT}}{V^2} = 0 \Rightarrow \gamma = \frac{3}{T}$.

Key (C)

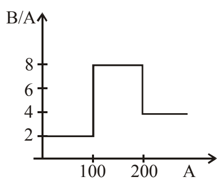
SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

30. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice (s) given below.

Figure



(A) fusion of two nuclei with mass numbers lying in the range of 1 < A < 50 will release energy
(B) fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy
(C) fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments
(D) fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

Sol (Energy released) = Total B.E. of products – Total B.E. of reactants.
Key (B) and (D)

31. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is not external force acting on the balls. Let $\vec{p}_1^{\,'}$ and $\vec{p}_2^{\,'}$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non–zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{p}_1^{\,'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2^{\,'} = a_2\hat{i} + b_2\hat{j}$
(B) $\vec{p}_1^{\,'} = c_1\hat{k}$, $\vec{p}_2^{\,'} = c_2\hat{k}$
(C) $\vec{p}_1^{\,'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2^{\,'} = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$
(D) $\vec{p}_1^{\,'} = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2^{\,'} = a_2\hat{i} + b_1\hat{j}$.

Sol Initial momentum of the system $= \vec{0}$.
In the absence of external forces, final momentum of the system must also be zero.
i.e., $\vec{P}_1^1 + \vec{P}_2^1 = \vec{0}$
Key (A) and (D)

32. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s),
(A) if d = λ, the screen will contain only one maximum
(B) if λ< d < 2λ, at least one more maximum (besides the central maximum) will be observed on the screen
(C) if the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(D) if the intensity of light falling on slit 2 is reduced so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase.

Sol. Condition for maxima is
$d \sin\theta = n\lambda$
If $d = \lambda$, $\sin\theta = n$
Possible value is 0
∴ only one maxima will be obtained
∴ (A) is correct.
IF $\lambda < d < 2\lambda$

$\Rightarrow \lambda < \frac{n\lambda}{\sin\theta} < 2\lambda$

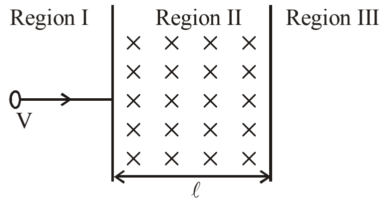
$\Rightarrow 1 < \frac{n}{\sin\theta} < 2 \Rightarrow n = 0, \pm 1.$

Key ∴ (B) is correct.

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ & $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

with $I_1 = 4I_2$
$I_{max} = 9I_2$, $I_{min} = I_2$
but when $I_1 = I_2$
$I_{max} = 4I_2$ and $I_{min} = 0$
(C) and (D) are wrong.

33. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s)
Figure :



(A) the particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

(B) the particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) time spent in Region II is same for any velocity V as long as the particle returns to Region I.

Sol. The radius of the circular path in region II $r = \frac{mV}{qB}$

If $r > \ell$, the particle enters the region III $\Rightarrow V > \frac{qB\ell}{m}$

$\therefore$ (A) is correct.
The path length will be maximum if it is able to describe a semi–circle

i.e., $\ell = r \Rightarrow V = \frac{q\ell B}{m}$

$\therefore$ (C) is correct.

If it is to return to region I, it must trace a semi–circle, the time for which is dependent of V $\left(T = \frac{\pi m}{qB}\right)$

Key **(D)**

SECTION – III
Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

34. STATEMENT – 1
Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.
and
STATEMENT – 2
By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol The acceleration of an object down an incline of angle θ is

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$$

Now, $I_{hollow} > I_{solid}$ for same mass and dimensions

$\therefore$ $a_{hollow} < a_{solid}$

$\therefore$ solid cylinder will reach the bottom first. $\therefore$ Statement–1 is false.

Key **(D)**

35. STATEMENT – 1

The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT – 2

In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol. As the water stream moves up, its speed decreases (due to gravity) and since flow rate (= Av) remains constant, the area increases making it spread like a fountain. The reverse is true when it moves down.

Both Statements are correct and Statement–2 is a correct explanation of Statement–1.

Key **(A)**

36. STATEMENT – 1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT – 2

Resistance of a metal increases with increase in temperature.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol In a meter bridge

$$\frac{R_X}{R_S} = \frac{X}{100 - X} \text{ at null point}$$

When unknown resistance is put in an enclosure maintained at a higher temperature, R_X will increase. To keep the null point same, R_S has to be increased.

$\therefore$ Statement–1 is false.

Key **(D)**

37. STATEMENT – 1

An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT – 2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of free–fall.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol. Statement–1 is correct and so is Statement–2. Both the astronaut and space station are in a state of free fall.

Key **(A)**

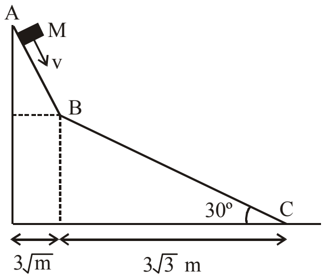
SECTION – IV
Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 38 to 40

A small block of mass M moves on a frictionless surface of an inclined plane, as show in figure. The angle of the incline suddenly changes from 60º to 30º at point B. The block is initially at rest at A. Assume that collision between the block and the incline are totally inelastic ($g = 10\ m/s^2$).

Figure :



38. The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s

(C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s.

Sol At B before collision $v_0 = \sqrt{60}$ m/s at an angle of 30º with downward vertical.

Hence component of v_0 along BC = $v_0 \cos 30º$

$\therefore$ impact force acts along the normal to the plane, hence $v_0 \cos 30º$ will not change and component of v_0 perpendicular to the plane becomes zero as collision is completely inelastic.

Key **(B)**

39. The speed of the block at point C immediately before it leaves the second incline is

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s

(C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

Sol. Potential Energy + Kinetic Energy = constant

$$\Rightarrow \frac{1}{2}m(v_0\cos 30°)^2 + mg \times 3m = \frac{1}{2}mv_e^2 \Rightarrow v_c = \sqrt{105}\ m/s.$$

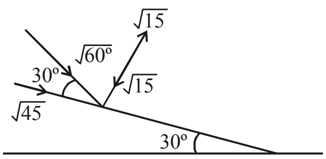
Key **(B)**

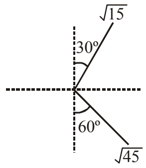
40. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s

(C) 0 (D) $-\sqrt{15}$ m/s.

Sol. Velocity after collision





$$U_y = \sqrt{15}\cos 30° - \sqrt{45}\cos 60°$$

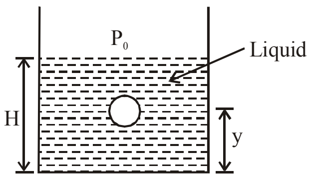
$$= \frac{\sqrt{45}}{2} - \frac{\sqrt{45}}{2} = 0.$$

Key **(C)**

Paragraph for Question Nos. 41 to 43

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with eth liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure :



41. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it
(A) only the force of gravity
(B) the force due to gravity and the force due to the pressure of the liquid
(C) the force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
(D) the force due to gravity and the force due to viscosity of the liquid.

Sol Besides the buoyancy force, the other forces are
(i) force of gravity, and
(ii) force due to viscosity
The buoyant force is due to the pressure of the liquid.

Key **(D)**

42. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{\frac{2}{5}}$ (B) $T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{\frac{2}{5}}$

(C) $T_0\left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{\frac{3}{5}}$ (D) $T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{\frac{3}{5}}$.

Sol For the gas inside the bubble, the adiabatic equation $T^Y P^{1-Y}$ = constant applies

$$\Rightarrow T_0^{\frac{5}{3}}(P_0 + \rho_\ell gH)^{-\frac{2}{3}} = T^{\frac{5}{3}}\{P_0 + \rho_\ell g(H-y)\}^{-\frac{2}{3}}$$

$$\Rightarrow T = \frac{T_0\{P_0 + \rho_\ell g(H-y)\}^{\frac{2}{5}}}{(P_0 + \rho_\ell gH)^{\frac{2}{5}}}$$

Key (B) is correct.

43. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0 \frac{(P_0 + \rho_\ell gH)^{\frac{2}{5}}}{(P_0 + \rho_\ell gy)^{\frac{7}{5}}}$ (B) $\frac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell gH)^{\frac{2}{5}}]P_0 + \rho_\ell g(H-y)]^{\frac{3}{5}}}$

(C) $\rho_\ell nRgT_0 \frac{(P_0 + \rho_\ell gH)^{\frac{3}{5}}}{(P_0 + \rho_\ell gy)^{\frac{8}{5}}}$ (D) $\frac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell gH)^{\frac{3}{5}}[P_0 + \rho_\ell g(H-y)]^{\frac{2}{5}}}$.

Sol Buoyancy force = $V\rho_\ell g$

where V is the volume of the bubble (at location y)

Now $V_0 = \frac{nRT_0}{(P_0 + \rho_\ell gH)}$

and PV^{Y} = constant

$$\Rightarrow \quad (P_0 + \rho_\ell gH)\left\{\frac{nRT_0}{P_0 + \rho_\ell gH}\right\}^{Y} = \{P_0 + \rho_\ell(H - y)\}V^{\gamma}$$

$$\Rightarrow \quad V = nRT_0\frac{(P_0 + \rho_\ell H)^{\frac{1-\gamma}{\gamma}}}{\{P_0 + \rho_\ell g(H - y)\}^{\frac{1}{\gamma}}}$$

$\therefore$ Buoyant force = $V\rho_\ell g$

Key **(B)**

Paragraph for Question Nos. 44 to 46

In a mixture of H – He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

44. The quantum number n of the state finally populated in He^+ ions is

(A) 2 (B) 3

(C) 4 (D) 5.

Sol. The excitation energy of the H atoms

= – 3.4 – (– 13.6) = 10.2 eV

Energy of the He^+ ions in their first excited state

= – 3.4 × 4 = – 13.6 eV

Energy after transferrence of energy

= – 13.6 + 10.2 = – 3.4 eV.

Now let n be the quantum number of the final state of He^+ ions, then

$$-3.4 = -13.6\frac{(2)^2}{n^2}$$

$\Rightarrow$ n = 4.

Key (C)

45. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is

(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m

(C) 4.8×10^{-7} m (D) 4.0×10^{-7} m.

Sol. $\Delta E = E_3 - E_4$

$$\Rightarrow \quad \frac{hc}{\lambda} = E_0 \times 4 \times \frac{7}{144}$$

$$\lambda = \frac{12.4 \times 10^{-7}\,\text{eVm} \times 36}{7 \times 13.6\,\text{eV}} \approx 4.8 \times 10^{-7}\,\text{m}$$

Key (C)

46. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 1 (D) 2.

Sol Kinetic energy of n = 2 electron for H atom = 3.4 eV

Kinetic energy of n = 2 electron for He^+ ion = 3.4 × 4 eV

$\therefore$ ratio = $\frac{1}{4}$

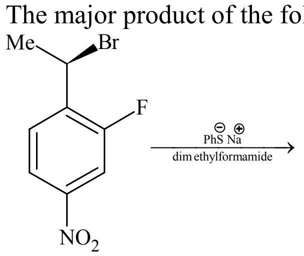
Key (A)

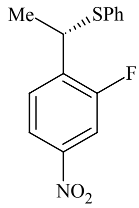
CHEMISTRY

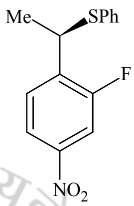
SECTION – I
Straight Objective Type

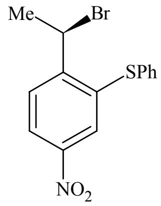
This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

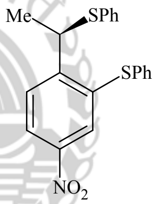
47. The major product of the following reaction is



(A)


(B)


(C)


(D)


Sol In DMF solvent S_N1 being not possible, S_N2 takes place resulting into inversion of configuration.
Key **(A)**

*48. Hperconjugation involves overlap of the following orbitals
(A) σ - σ (B) σ - p
(C) p – p (D) π - π

Sol Hyperconjuation is σ - π, σ - odd e^- and σ - cationic carbon conjugation and it results into π bond formation by p – p overlap.
Key **(C)**

49. Aqueous solution of $Na_2S_2O_3$ on reaction with Cl_2 gives
(A) $Na_2S_4O_6$ (B) $NaHSO_4$
(C) NaCl (D) NaOH

Sol $Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$
Key **(B)**

50. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of
(A) nitrogen (B) oxygen
(C) carbon dioxide (D) argon

Sol $Ag + NaCN + O_2 + H_2O \longrightarrow Na[Ag(CN)_2] + NaOH$
Key **(B)**

*51. 2.5 mL of $\frac{2}{5}$ M weak monoacidic base ($K_b = 1 \times 10^{-12}$ at 25°C) is titrated with $\frac{2}{15}$ M HCl in water at 25°C. the concentration of H^+ at equivalence point is ($K_w = 1 \times 10^{-14}$ at 25°C)

(A) 3.7×10^{-13}M
(B) 3.2×10^{-7}M
(C) 3.2×10^{-2} M
(D) 2.7×10^{-2}M

Sol 0.4M BOH ($K_b = 10^{-12}$) + $\frac{2}{15}$MHCl

$pH = \frac{1}{2}[pK_w - pK_b - \log c] = \frac{1}{2}[14 - 12 - \log 0.1]$

$= 1.5$, $[H^+] = 10^{-1.5} = 10^{0.5} \times 10^{-2}$

$= 3.2 \times 10^{-2}$M

Key **(C)**

52. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reaction is

(A) 0.5 mol^{-1} dm^3
(B) 1.0 mol dm^{-3}
(C) 1.5 mol dm^{-3}
(D) 2.0 mol^{-1} dm^3

Sol $K_1 = \frac{0.693}{40}$

$K_0 = \frac{1.386}{2 \times 20} = \frac{1.386}{40} \min^{-1}$

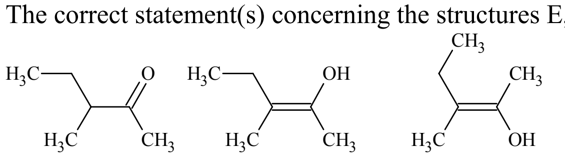
$\frac{K_1}{K_0} = \frac{0.693}{1.386} = \frac{1}{2}$

Key **(A)**

SECTION – II
Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

*53. The correct statement(s) concerning the structures E, F and G is (are)



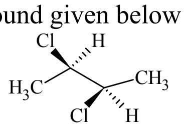
(E) **(F)** **(G)**

(A) E, F and G are resonance structures
(B) E, F and E, G are tautomers
(C) F and G are geometrical isomers
(D) F and G are diastereomeers

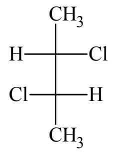
Sol F is the enol form of E and so is also G for E. F and G are also geometrical isomers.

Key **(B), (C), (D)**

54. The correct statement(s) about the compound given below is (are)



(A) The compound is optically active
(B) The compound possesses centre of symmetry
(C) The compound possesses plane of symmetry
(D) The compound possesses axis of symmetry

Sol  Two chiral centres are of same configurations and hence optically active.

Key **(A)**

*55. A gas described by van der Waals equation

(A) behaves similar to an ideal gas in the limit of large molar volumes
(B) behaves similar to an ideal gas in the limit of large pressure
(C) is characterized by van der Waals coefficients that are dependent on the identity of the gas but are independent of the temperature
(D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally

Sol $\left(P+\frac{a}{V^2}\right)(V-b)=RT$

When V is very large V – b $\approx$ V and $P+\frac{a}{V^2}\approx P,$ so PV = RT (for mole of an ideal gas). Due to inward pull acting on molecule striking the wall, the pressure decreases. The van der Waal's constants "a" and "b" are characteristics of a gas and as per van der Waal's they are temperature independent.

Key **(A), (C), (D)**

56. A solution of colourless salt H on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt(s) H is (are)

(A) NH_4NO_3 (B) NH_4NO_2
(C) NH_4Cl (D) $(NH_4)_2SO_4$

Sol $NH_4X + NaOH \longrightarrow NaX + H_2O + NH_3\uparrow$

$(X^- = NO_3^-, NO_2^-, Cl^-, SO_4^{2-})$

$NO_3^- + 4Zn + 7OH^- \longrightarrow 4ZnO_2^{-2} + 2H_2O + NH_3\uparrow$

$NO_2^- + 3Zn + 5OH^- \longrightarrow 3ZnO_2^{-2} + H_2O + NH_3$

Key **(A), (B)**

SECTION – III
Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

*57. STATEMENT-1
The plot of atomic number (y-axis versus number of neutrons (x-axis) for stable nuclei shows a curvature towards x-axis from the line of 45° slope as the atomic number is increased.
and
STATEMENT-2:
Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons in heavier nuclides.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol When Z > 20, the number of neutron must increase above the number of protons so as to overcome proton-proton repulsion i.e., $\frac{n}{p} > 1$.

Key **(A)**

*58. STATEMENT-1
For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.
and
STATEMENT-2
At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol G is minimum while $\Delta G = 0$ at equilibrium
Key **(D)**

59. STATEMENT-1
Bromobenzene upon reaction with Br_2/Fe gives 1, 4-dibromobenzeen as the major product.
and
STATEMENT-2
In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile..
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol In bromobenzene, inductive effect is responsible for deactivating the benzene nucleus and has no effect on directive influence. The directive influence is governed solely by mesomeeric effect.
Key **(C)**

*60. STATEMENT-1
Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.
and
STATEMENT-2
The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to 'inert pair effect'.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol Sn and Pb both have ns^2np^2 configurations of their valence shells. Moving down a group, the ns electron pair becomes more and more inert towards bonding called inert pair effect. This is maximum in Pb. So Pb^{4+} tends to get reduced to Pb^{2+} i.e., Pb^{4+} is stronger oxidizing agent then Sn^{4+} in which the inert pair effect is relatively less.
Key **(C)**

SECTION – IV
Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 61 to 63

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when solute molecules are added to get homogeneous solution. These are called colligative properties. Applications of colligative properties are very useful in day-to-day life. One of its examples is the use of ethylene glycol and water mixtures as anti-freezing liquid in the radiator of automobiles.

A solution **M** is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture is 0.9

Given: Freezing point depression constant of water $(K_f^{water}) = 1.86$ K kg mol^{-1}
Freezing point depression constant of ethanol $(K_f^{ethanol}) = 2.0$ K kg mol^{-1}
Boiling point elevation constant of water $(K_b^{water}) = 0.52$ K kg mol^{-1}
Boiling point elevation constant of ethanol $(K_b^{ethanol}) = 1.2$ K kg mol^{-1}
Standard freezing point of water = 273K
Standard freezing point of ethanol = 155.7 K
Standard boiling point of water = 373 K
Standard boiling point of ethanol = 351.5 K
Vapour pressure of pure water = 32.8 mm Hg
Vapour pressure of pure ethanol = 40 mm Hg
Molecular weight of water = 18 g mol^{-1}
Molecular weight of ethanol = 46 g mol^{-1}

In answering the following questions, consider the solutions to be ideal dilute solutions and solutes to be non-volatile and non-dissociative.

61. The freezing point of the solution **M** is
(A) 268.7 K (B) 268.5 K
(C) 234.2 K (D) 150.9 K

Sol 0.1 mole water in 0.9 mol i.e. 0.9 × 46g ethanol

$\therefore$ molality of water in the solution $= \frac{0.1\times 1000}{0.9\times 46} = \frac{100}{41} = 2.5$

$\Delta T_f = k_f \cdot C_m = 2\times 2.5 = 5$

T_f of solution = 155.7 – 5 = 150.7K

Key (D)

62. The vapour pressure of the solution **M** is
(A) 39.3 mm Hg (B) 36.0 mm Hg
(C) 29.5 mm Hg (D) 28.8 mm Hg

Sol $p_{mix} = x_1 p_1^0 + x_2 p_2^0$
= 0.1 × 32,5 + 0.9 × 40
= 3.28 + 36 = 39.28 mm

Key (A)

63. Water is added to the solution M such that the mole fraction of water in the solution becomes 0.9 The boiling point of this solution is
(A) 380.4 K (B) 376.2 K
(C) 373.5 K (D) 354.7 K

Sol $x_{C_2H_5OH} = 0.1,\ \ x_{H_2O} = 0.9$

18 × 0.9g of H_2O has 0.1 mol of C_2H_5OH

$\therefore$ molality of $C_2H_5OH = \dfrac{0.1}{18\times0.9}\times1000 = \dfrac{100}{16.2}$ mole kg^{-1}

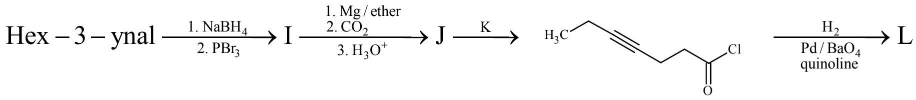
$\therefore\ \Delta T_b = 0.52\times\dfrac{100}{16.2} = \dfrac{52}{16.2} = 3.2\,K$

$\therefore$ boiling point = 373 + 3.2 = 376.2K

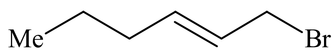
Key **(B)**

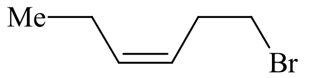
Paragraph for Questions Nos. 64 to 66

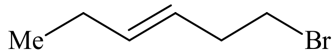
In the following reaction sequence product I, J and L are formed. K represents a reagent.

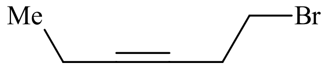


64. The structure of the product I is:

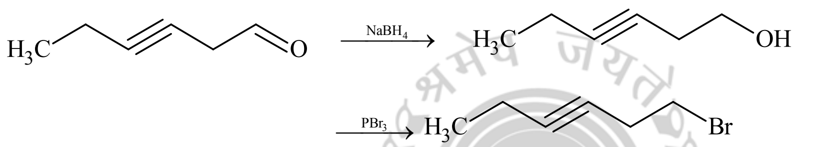
(A) 

(B) 

(C) 

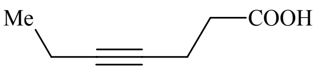
(D) 

Sol

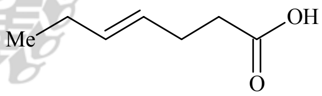


Key **(D)**

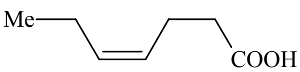
65. The structures of compound J and K, respectively, are

(A) 

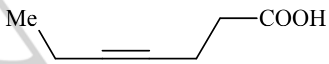
and $SOCl_2$

(B) 

and SO_2Cl_2

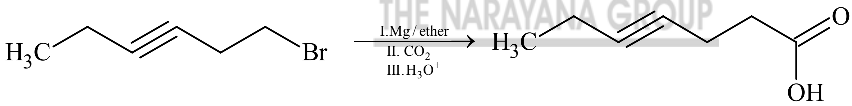
(C) 

and $SOCl_2$

(D) 

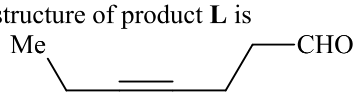
and CH_3SO_2Cl

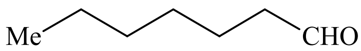
Sol

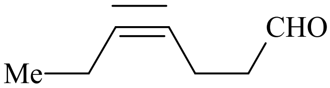


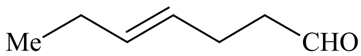
Key **(A)**

66. The structure of product **L** is

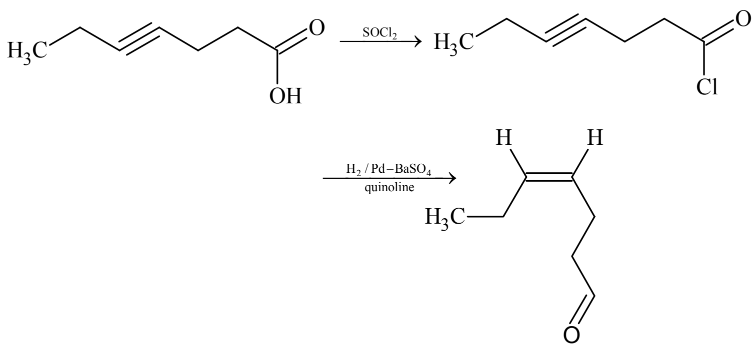
(A) 

(B) 

(C) 

(D) 

Sol



Key (C)

Paragraph for Questions Nos. 67 to 69

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water. Nitrates re difficult to reduce under the laboratory conditions but microbes do it easily. Ammonia forms large number of complexes with transition metal ions. Hybridization easily explains the ease of sigma donation capability of NH_3 and PH_3. Phosphine is a flammable gas and is prepared from white phosphorous.

67. Among the following, the correct statement is
(A) Phosphates have no biological significance in humans
(B) Between nitrates and phosphates, phosphates are less abundant in earth's crust
(C) Between nitrates and phosphates, nitrates are less abundant in earth's crust
(D) Oxidation of nitrates is possible in soil.

Sol Nitrates being water soluble and as they get reduced by microbes so obviously its abundance will decrease.

Key **(C)**

*68. Among the following, the correct statement is
(A) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional
(B) Between NH_3 and PH_3, PH_3 si a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(C) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(D) Between NH_3 and PH_3, PH_3 is a better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional.

Sol NH_3 is a stronger Lewis base than PH_3.

Key (C)

69. White phosphorus on reaction with NaOH gives PH_3 as one of the products. This is a
(A) dimerization reaction (B) disproportionation reaction
(D) condensation reaction (D) precipitation reaction

Sol $\overset{0}{\underline{P}}_4 + 3NaOH + 3H_2O \longrightarrow 3NaH_2\overset{+1}{\underline{P}}O_2 + \overset{-3}{\underline{P}}H_3$

Here P_4 is oxidized to NaH_2PO_2 and it is reduced to PH_3.

Key (B)

MATHEMATICS

PAPER - II

SECTION – I
Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Let the function g : $(-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is

(A) even and is strictly increasing in $(0, \infty)$
(B) odd and is strictly decreasing in $(-\infty, \infty)$
(C) odd and is strictly increasing in $(-\infty, \infty)$
(D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

Sol $g(u) = 2\tan^{-1}(e^u) - \pi/2$

$g'(u) = \frac{2e^u}{1+e^{2u}} > 0$

$g(-u) = 2\tan^{-1}(e^{-u}) - \pi/2$

$\Rightarrow g(-u) = -g(u)$

Key **(C)**

*2. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\pi/2$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(A) 6 + 7i
(B) -7 + 6i
(C) 7 + 6i
(D) -6 + 7i

Sol $z_1 = (6 + 2\hat{i})$

$z'_1 = (6 + 5\hat{i})$

$\vec{r}_1' = 6\hat{i} + 5\hat{j}$

$\vec{d}_1 = \hat{i} + \hat{j}$

$\vec{r}_2' = 7\hat{i} + 6\hat{j}$

$z_2' = (7 + 6\hat{i})$

$z_2 = (7 + 6i)\,i = -6 + 7i$

Key **(D)**

*3. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{\frac{2}{3}}$
(B) $\sqrt{\frac{3}{2}} - 1$
(C) $1 + \sqrt{\frac{2}{3}}$
(D) $\sqrt{\frac{3}{2}} + 1$

Sol $x^2 - 2\sqrt{2}x - 2(y^2 + 2\sqrt{2}y) = 6$

$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 - 2 + 4 = 6$

$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$

$\Rightarrow \dfrac{(x-\sqrt{2})^2}{4} - \dfrac{(y+\sqrt{2})^2}{2} = 1$

$b^2 = a^2 (e^2 - 1)$

$\Rightarrow 2 = 4 (e^2 - 1) \Rightarrow e^2 - 1 = 1/2$

$e = \sqrt{3}/2$

area = $\dfrac{1}{2}$ (ae – a) × b^2/a = (e – 1) = $\left(\sqrt{\dfrac{3}{2}} - 1\right)$

Key **(B)**

4. Let I = $\displaystyle\int \frac{e^x}{e^{4x}+e^{2x}+1}dx$, J = $\displaystyle\int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. Then, for an arbitrary constant C, the value of J – I equals

(A) $\dfrac{1}{2}\log\left(\dfrac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right) + C$

(B) $\dfrac{1}{2}\log\left(\dfrac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right) + C$

(C) $\dfrac{1}{2}\log\left(\dfrac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right) + C$

(D) $\dfrac{1}{2}\log\left(\dfrac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right) + C$

Sol J-I = $\displaystyle\int\left(\frac{e^{-x}}{e^{-4x}+e^{-2x}+1} - \frac{e^x}{e^{4x}+e^{2x}+1}\right)dx = \int \frac{e^x(e^{2x}-1)}{e^{4x}+e^{2x}+1}dx$. Put $e^x = t$

$\displaystyle\int \frac{t^2-1}{t^4+t^2+1}dt = \frac{1}{2}\log\left|\frac{\frac{t+1}{t}-1}{\frac{t+1}{t}+1}\right| + c = \frac{1}{2}\log\left|\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right| + c$

Key **(C)**

5. Let Two non-collinear unit vector $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. Where P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$

(B) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$

(C) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + 2\,\hat{a}.\hat{b})^{1/2}$

(D) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + 2\,\hat{a}.\hat{b})^{1/2}$

Sol $\overline{OP} = \left|\bar{a}\cos t + \bar{b}\sin t\right| = \sqrt{1+\bar{a}.\bar{b}\sin 2t} \le \sqrt{1+\bar{a}.\bar{b}}$

$M = \sqrt{1+\bar{a}.\bar{b}}$

Then $\bar{u} = \dfrac{\bar{a}+\bar{b}}{|\bar{a}+\bar{b}|}$

Key **(A)**

6. Let g(x) = logf(x) where f(x) is a twice differentiable positive function on (0, ∞) such that f(x + 1) = xf(x). Then, for N = 1, 2, 3, ... ,

$g''\left(N+\dfrac{1}{2}\right) - g''\left(\dfrac{1}{2}\right) =$

(A) $-4\left\{1+\dfrac{1}{9}+\dfrac{1}{25}+...+\dfrac{1}{(2N-1)^2}\right\}$

(B) $4\left\{1+\dfrac{1}{9}+\dfrac{1}{25}+...+\dfrac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1+\dfrac{1}{9}+\dfrac{1}{25}+...+\dfrac{1}{(2N+1)^2}\right\}$

(D) $4\left\{1+\dfrac{1}{9}+\dfrac{1}{25}+...+\dfrac{1}{(2N+1)^2}\right\}$

Sol $g(x) = \log f(x)$
$g(x+1) = \log f(x+1)$
$= \log x f(x)$
$= \log x + \log f(x)$
$g(x+1) = \log x + g(x)$
$\Rightarrow g(x+1) - g(x) = \log x$
$g'(x+1) - g'(x) = 1/x$

$$g''(x+1) - g''(x) = \frac{-1}{x^2}$$

$$g''\left(x+\frac{1}{2}\right) - g''\left(x-\frac{1}{2}\right) = \frac{-1}{\left(x-\frac{1}{2}\right)^2} = \frac{-2^2}{(2x-1)^2}$$

$$g''(3/2) - g''\left(\frac{1}{2}\right) = \frac{-2^2}{1^2} \qquad \text{... (i)}$$

$$g''(5/2) - g''(3/2) = \frac{-2^2}{3^2} \qquad \text{... (ii)}$$

$$g''(7/2) - g''(5/2) = \frac{-2^2}{5^2}$$

$$g''\left(N+\frac{1}{2}\right) - g''\left(N-\frac{1}{2}\right) = \frac{-2^2}{(2N-1)^2}$$ (By adding the above) we have

$$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -2^2\left[1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots+\frac{1}{(2N-1)^2}\right]$$

Key **(A)**

7. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ (B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ (D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

Sol $$A = \int_0^{\pi/4}\left(\sqrt{\frac{1+\tan x/2}{1-\tan x/2}} - \sqrt{\frac{1-\tan x/2}{1+\tan x/2}}\right)dx$$

$$= \int_0^{\pi/4} \frac{2\tan x/2}{\sqrt{1-\tan x/2}}dx = \int_0^{\tan\pi/8} \frac{4tdt}{(1+t^2)\sqrt{1-t^2}}$$

Key **(B)**

8. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8 (B) 3, 6 or 9
(C) 4 or 8 (D) 5 or 10

Sol $S = \{x_1, x_2, \ldots x_{10}\}$
A = {- - - - }
$n(B) = x$

$$P(A) = \frac{4}{10}$$

$P(B) = \frac{x}{10}$

$P(A \cap B) = \frac{y}{10}$

$\frac{4x}{100} = \frac{y}{10}$

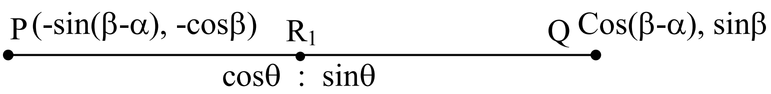
$4x = 10y \Rightarrow x = 5, 10$

Key **(D)**

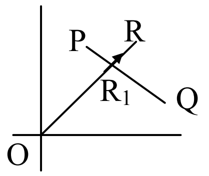
*9. Consider three points $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \pi/4$. Then,

(A) P lies on the line segment RQ
(B) Q lies on the line segment PR
(C) R lies on the line segment QP
(D) P, Q, R are non-collinear

Sol



$$R_1 \equiv \left(\frac{-\sin(\beta-\alpha)\sin\theta + \cos x(\beta-\alpha)\cos\theta}{\cos\theta+\sin\theta}, \frac{-\cos\beta\sin\theta+\cos\theta\sin\beta}{\cos\theta+\sin\theta}\right) = \left(\frac{\cos(\beta-\alpha+\theta}{\cos\theta+\sin\theta}, \frac{\sin(\beta-\theta)}{\cos\theta+\sin\theta}\right)$$



$\overline{OR} = (\cos\theta + \sin\theta)\ \overline{OR} \Rightarrow$ P, Q, R are non-collinear

as $1 < \cos\theta + \sin\theta < \sqrt{2}$

Key **(D)**

SECTION – II

Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

*10. Consider

$L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0$,

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$

STATEMENT–1

If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

STATEMENT–2

If Line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol $\frac{|p+3-p+3|}{\sqrt{2^2+3^2}} = \frac{6}{\sqrt{13}} < 2$

⇒ distance between parallel lines is less then the radius of the circle
⇒ If one is chord then other may or may not be a diameter
and if one is diameter other must be a chord.

Key **(C)**

11 Let a solution y = y(x) of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$

STATEMENT–1

$y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$

and

STATEMENT–2

y(x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0,\ y(2) = \frac{2}{\sqrt{3}}$

$\int\frac{dy}{y\sqrt{y^2-1}} = \int\frac{dx}{x\sqrt{x^2-1}} \Rightarrow \sec^{-1}y = \sec^{-1}x + c$

$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \sec^{-1}(2) + c$

$\pi/6 = \pi/3 + c \Rightarrow c = -\pi/6$

$\sec^{-1}y = \sec^{-1}x - \pi/6 \Rightarrow y = \sec(\sec^{-1}x - \pi/6)$

So statement 1 is correct.

Now $\frac{1}{y} = \cos(\sec^{-1}x - \pi/6) = \frac{1}{x}\frac{\sqrt{3}}{2} + \frac{\sqrt{x^2-1}}{x}\frac{1}{2} \Rightarrow \frac{1}{y} = \frac{\sqrt{3}}{2x} + \frac{1}{2}\sqrt{1-\frac{1}{x^2}}$

Statement 2 is false.

Key **(C)**

*12 Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT–1

The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

STATEMENT–2

The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol Let $a_1 = a$, $a_2 = a_5$, $a_3 = ar^2$, $a_4 = ar^3$

$b_1 = a$, $b_2 = a + ar$, $b_3 = a + ar + ar^2$, $b_4 = a + ar + ar^2 + ar^3$. So b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. nor in H.P. So, statement-1 is true and statement 2 is false.

Key **(C)**

*13 Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

STATEMENT–1: $(p^2 - q)(b^2 - ac) \ge 0$

and

STATEMENT–2: $b \ne pa$ or $c \ne qa$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol $a, b, c, p, q \in R$

If $\alpha \notin R$, then $\beta = \bar{\alpha} = \frac{1}{\beta} \Rightarrow \beta^2 = 1$ not possible

SO, $\alpha, \beta \in R$

$\Rightarrow D_1 \ge 0$, $D_2 \ge 0 \Rightarrow$ statement-I is correct.

Now, let $b = pa$ and $c = qa$

$\Rightarrow ax^2 + 2bx + c \equiv a(x^2 + 2px + q)$

$\Rightarrow$ Both roots are common which is not the case

Hence our supposition is wrong.

$\Rightarrow b \ne pa$ or $c \ne qa$

Now consider statement : 2

$\Rightarrow$ both roots are not common

$\Rightarrow$ exactly one root is common or No root common

(but as per paragraph α is a common root)

$\Rightarrow$ exactly one root is common.

$\Rightarrow$ common root must be real

$\Rightarrow$ both equation has real roots.

Key **(A)**

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 14 to 16

Consider the function $f : (-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, 0 < a < 2$$

14. Which of the following is true?

(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$
(B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$
(C) $f'(1) f'(-1) = (2 - a)^2$
(D) $f'(1) f'(-1) = -(2 + a)^2$

15. Which of the following is true?

(A) f(x) is decreasing on (–1, 1) and has a local minimum at x = 1
(B) f(x) is increasing on (–1, 1) and has a local maximum at x = 1
(C) f(x) is increasing on (–1, 1) but has neither a local maximm nor a local minimum at x = 1
(D) f(x) is decreasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1

16. Let

$$g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2}\,dt$$

Which of the following is true?

(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
(B) $g'(x)$ is negative no $(-\infty, 0)$ and positive on $(0, \infty)$
(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
(D) $g'(x)$ does not change sign on $(-\infty, \infty)$

Sol 14-16

14. $f(x) = 1 - \dfrac{2ax}{x^2 + ax + 1}$

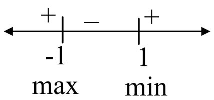
$$\Rightarrow \; f'(x) = \frac{2a(x^2-1)}{(x^2+ax+1)^2}$$

$$\Rightarrow \; f''(x) = \frac{(x^2+ax+1)\left[(x^2+ax+1)4ax - 4a(x^2-1)(2x+a)\right]}{(x^2+ax+1)^4}$$

$f''(1) = 4a / (2 + a)^2$
$f''(-1) = -4a/ (2 - a)^2$

Key (A)

15. $f'(x) = \dfrac{2a(x^2-1)}{(x^2+ax+1)^2}$



So f(x) is decreasing in (-1, 1)
and local minima at x = 1

Key (A)

16. $g(x) = \displaystyle\int_0^{e^x} \frac{f'(t)}{1+t^2}$

$$g'(x) = \frac{f'(e^x)}{1+(e^x)^2}e^x$$

$$f'(e^x) = \frac{2a\,(e^{2x}-1)}{(e^{2x}+ae^x+1)^2}$$

Key : (B)

Paragraph for Questions Nos. 17 to 19

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

17. The unit vector perpendicular to both L_1 and L_2 is

(A) $\dfrac{-\hat{i} + 7\hat{j} + 7\hat{k}}{\sqrt{99}}$ (B) $\dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

(C) $\dfrac{-\hat{i} + 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$ (D) $\dfrac{7\hat{i} - 7\hat{j} - \hat{k}}{\sqrt{99}}$

18. The shortest distance between L_1 and L_2 is

(A) 0 (B) $\frac{17}{\sqrt{3}}$

(C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

19. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$

(C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

Sol 17-19

17. Line L_1 is parallel to vector $3\hat{i}+\hat{j}+2\hat{k}$ $(\vec{n}_1)$. line L_2 is parallel to vector $\hat{i}+2\hat{j}+3\hat{k}\,(\vec{n}_2)$

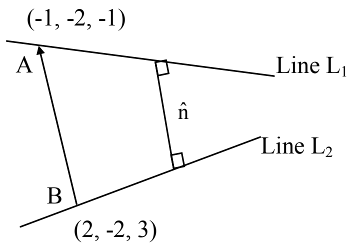
unit vector perpendicular to both L_1 and L_2 is $\frac{\vec{n}_1\times\vec{n}_2}{|\vec{n}_1\times\vec{n}_2|}$

Now, $\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ 1 & 2 & 3\end{vmatrix} = -\hat{i}-7\hat{j}+5\hat{k}$

$\therefore$ Required unit vector ($\hat{n}$) = $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

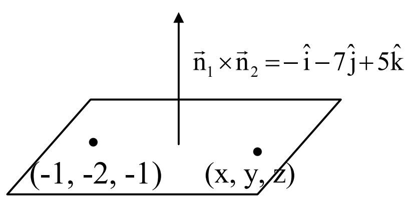
Key (B)

18. S.D. = $\left|\overline{BA}.\hat{n}\right| = \left|(-3\hat{i}-4\hat{k}).\frac{(-\hat{i}-7\hat{j}+5\hat{k})}{5\sqrt{3}}\right| = \frac{17}{5\sqrt{3}}$



Key (D)

19.



Equation of plane is
$(x + 1)(-1) + (y + 2)(-7) + (z + 1)5 = 0$
or, $x + 7y - 5z + 10 = 0$ ... (1)

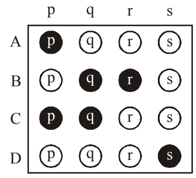
Distance of point (1, 1, 1) from the plane (1) is $\frac{13}{\sqrt{75}}$

Key (C)

SECTION – IV

Matrix Match Type

*This section contains 3 questions. Each question contains statements given in two column which have to be matched. Statements (A, B, C, D) in **Column I** have to be matched with statements (p, q, r, s) in **Column II**. The answers to these questions have to be appropriately bubbles as illustrated in the following example.*
If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows :



*20. Consider the lines given by

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

Match the statements/Expressions in Column I with the statements/Expressions in Column II. Indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I		**Column II**
(A)	L_1, L_2, L_3 are concurrent, if	(p)	$k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q)	$k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(r)	$k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(s)	$k = 5$

Sol $x + 3y = 5$

$3x - ky = 1$

$5x + 2y = 12$

(A) $\begin{vmatrix} 1 & 3 & 5 \\ 3 & -k & 1 \\ 5 & 2 & 12 \end{vmatrix} = 0 \Rightarrow 13k - 65 = 0 \Rightarrow k = 5$

(B) $L_1 \parallel L_2 \Rightarrow -\frac{1}{3} = \frac{3}{k} \Rightarrow k = -9$

or $L_2 \parallel L_3 \Rightarrow \frac{3}{k} = -5/2 \Rightarrow k = -6/5$

(C) no two should be parallel

and no three should be concurrent $\Rightarrow k = 5/6$

(D) k = -9, -6/5 , 5

Key (A) – (s)

(B) – (p), (q)

(C) – (r)

(D) – (p), (q), (s)

*21. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the entries in Column I with the correctly related quantum number(s) in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

	Column I		**Column II**
(A)	The number of permutations containing the word ENDEA is	(p)	5!
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	(q)	2 × 5!
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(r)	7 × 5!
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s)	21 × 5!

Sol $\boxed{\text{ENDEA}}\text{NOEL} \rightarrow E - 3, N - 2, D - 1, O - 1, A\ \text{-}1, L - 1$

(A) (ENDEA), N, O, E, L

$\Rightarrow 5!$

(B) $\frac{1}{E}$ - - - - - - - $\frac{1}{E}$ $\Rightarrow \frac{7!}{2!} = 21 \times 5!$

(C) - - - - $\boxed{-----}$

$\Rightarrow \frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$

(D) $\overline{1}\ \overline{2}\ \overline{3}\ \overline{4}\ \overline{5}\ \overline{6}\ \overline{7}\ \overline{8}\ \overline{9}$

There are 5 odd places 1, 3, 5, 7, 9 $\Rightarrow \frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

Key **(A) – (p)**
(B) – (s)
(C) – (q)
(D) – (q)

22. Match the statements/Expressions in Column I with the statements/Expressions in Column II. Indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I	**Column II**
*(A)	The minimum value of $\frac{x^2 + 2x + 4}{x + 2}$ is	(p) 0
(B)	Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and (A + B) (A – B) = (A – B) (A + B). If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
*(C)	Let a = $\log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k + 3^{-a})} < 2$, must be less than	(r) 2
*(D)	If sin θ = cos ϕ, then the possible values of $\frac{1}{\pi}(\theta \pm \phi - \frac{\pi}{2})$ are	(s) 3

Sol (A) Let $\frac{x^2 + 2x + 4}{x + 2} = y$

$\Rightarrow x^2 + (2 - y)x + 4 - 2y = 0$

$D \geq 0 \Rightarrow (y - 2)(y + 6) \geq 0$

$\Rightarrow y \in (-\infty, -6] \cup [2, \infty)$

Local min. = 2

In the problem in place of minimum, it should be local minimum

(B) $A' = A$ and $B' = -B$

and $(A + B)(A - B) = (A - B)(A + B) \Rightarrow BA = AB$

Now $(AB)' = B'A' = -BA = -AB$

$\Rightarrow$ k should be odd. $\Rightarrow k = 1, 3$

(C) $a = \log_3(\log_3 2)$

$3^{-a} = \log_2 3$

$\Rightarrow 2^{(-k + 3^{-a})} = 2^{-k} . 3$

$\Rightarrow \frac{1}{3} < 2^{-k} < \frac{2}{3}$

$\Rightarrow k = 1$

(D) $\sin\theta = \cos\phi$

$\Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) = \cos\phi$

$\Rightarrow \cos\left(\theta - \frac{\pi}{2}\right) = \cos\phi$

$\theta - \frac{\pi}{2} = 2n\pi \pm \phi$

$\Rightarrow \theta \mp \phi - \frac{\pi}{2} = 2n\pi$

$\frac{1}{\pi}\left(\theta \mp \phi - \frac{\pi}{2}\right) = 2n$

= even no. = 0, 2

Key **(A) – (r)**
(B) – (q), (s)
(C) – (q)
(D) – (p), (r)

PHYSICS

SECTION – I
Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

23. A radioactive sample S1 having an activity of 5μCi has twice the number of nuclei as another sample S2 which has an activity of 10 μCi. The half lives of S1 and S2 can be
(A) 20 years and 5 years, respectively
(B) 20 years and 10 years respectively
(C) 10 years each
(D) 5 years each.

Sol Activity $A = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot N \quad \therefore \quad \frac{(\ln 2)N_1}{T_1} = 5\mu Ci$

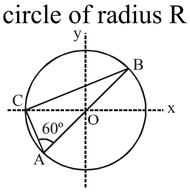
and $\frac{(\ln 2)N_2}{T_2} = 10\mu Ci$

But $N_1 = 2N_2$

$\therefore \quad 2 \cdot \frac{T_2}{T_1} = \frac{1}{2} \Rightarrow \frac{T_2}{T_1} = \frac{1}{4}$

Key **(A)**

24. Consider a system of three charges $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60º.



(A) the electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x–axis

(B) the potential energy of the system is zero

(C) the magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$

(D) the potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$.

Sol The electric field due to charges at A and B are equal and opposite. Therefore, at O, the electric field is only due to C, which has a magnitude $\frac{2q}{12\pi\varepsilon_0 R^2} = \frac{q}{6\pi\varepsilon_0 R^2}$.

$\therefore$ (A) is wrong.

The potential energy of the system

$$U = \sum \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r} \neq 0$$

$\therefore$ (B) is wrong.

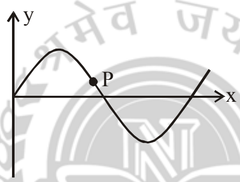
Force between C and B is $= \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{(2R\sin 60°)^2}$

$$= \frac{q^2}{54\pi\varepsilon_0 R^2}.$$

Key **(C)**

25. A transverse sinusoidal wave moves along a string in the positive x–direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap–shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is

Figure :



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

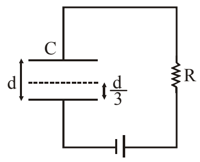
(C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s.

Sol $v_P = \frac{\partial y}{\partial t} = -v\frac{\partial y}{\partial x}$ is positive and can only be along y–axis.

Key **(A)**

26. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is

Figure :



(A) $\frac{6\varepsilon_0 R}{5d+3Vt}$ (B) $\frac{(15d+9Vt)\varepsilon_0 R}{2d^2-3dVt-9V^2t^2}$

(C) $\frac{6\varepsilon_0 R}{5d-3Vt}$ (D) $\frac{(15d-9Vt)\varepsilon_0 R}{2d^2-3dVt-9V^2t^2}$.

Sol $C = \frac{\frac{2\varepsilon_0}{x}\times\frac{\varepsilon_0}{d-x}}{\frac{2\varepsilon_0}{x}+\frac{\varepsilon_0}{d-x}}$ where $x = \frac{d}{3} - Vt$ $= \frac{2\varepsilon_0}{2d-x}$

The time constant, $\tau = RC = \dfrac{6\varepsilon_0 R}{5d + 3Vt}$

Key **(A) is correct.**

27. A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is

(A) 344 (B) 336
(C) 117.3 (D) 109.3.

Sol The wavelength λ corresponding to the first overtone of the air column is given by,

$\dfrac{3\lambda}{4} = 0.75\,\text{m}$

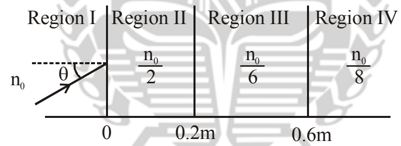
or $\lambda = 1$ m.

$\therefore \quad v = \dfrac{v}{\lambda} = 340$ Hz; the frequency of the tuning fork = 344 Hz

Key **(A) is correct.**

28. A light beam is travelling from Region I to Region IV (refer figure). The refractive index in Region I, II, III and IV are $n_0, \dfrac{n_0}{2}, \dfrac{n_0}{6}$ and $\dfrac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is

Figure :



(A) $\sin^{-1}\left(\dfrac{3}{4}\right)$ (B) $\sin^{-1}\left(\dfrac{1}{8}\right)$

(C) $\sin^{-1}\left(\dfrac{1}{4}\right)$ (D) $\sin^{-1}\left(\dfrac{1}{3}\right)$.

Sol In order that the beam not enter region IV, the angle θ should be less than a critical value so that :

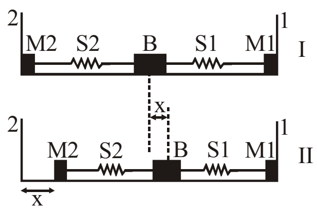
$n_0 \sin\theta_C = \dfrac{n_0}{8}\sin 90°$ i.e., $\theta_C = \sin^{-1}\left(\dfrac{1}{8}\right)$

Key **(B)**

29. A block (B) is attached to two unstretched springs S1 and S2 with spring constants k and 4k, respectively (see figure I). The other ends are attached to identical supports M1 and M2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measure with respect to the equilibrium position of the block B.

The ratio $\dfrac{y}{x}$ is

Figure :



(A) 4 (B) 2

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$.

Sol When the block undergoes a maximum displacement y towards wall 2, the spring S1 is at its natural length but S2 is compressed.

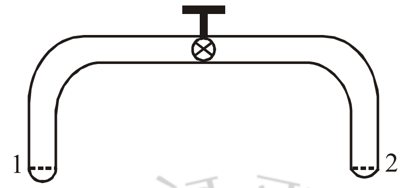
$\therefore \ \frac{1}{2}(4k)y^2 = \frac{1}{2}k.x^2$

or $y = \frac{x}{2}$

Key **(C)**

30. A glass tube of uniform internal radius (r) has valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub–hemispherical soap bubble as shown in figure. Just after opening the valve,

Figure :



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(C) no change occurs
(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases.

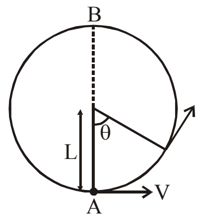
Sol The excess pressure inside a soap bubble $= \frac{4T}{R}$

Where R is the radius of curvature
Now radius of curvature at end 1 is less than the radius of curvature at end 2.

Key **(B)**

31. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies

Figure



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$.

Sol $V = \sqrt{5gL}$

Using, COE,

$\left(\frac{V}{2}\right)^2 = V^2 - 2gL(1-\cos\theta)$

$\frac{15}{8} = 1 - \cos\theta$ or $\cos\theta = \frac{-7}{8}$

Key **(D) is correct.**

SECTION – II
Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

32. STATEMENT – 1
For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
because
STATEMENT – 2
The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.
Sol. Both the statements are true and statement–2 explains statement–1.
Key **(A)**

33. STATEMENT – 1
The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.
because
STATEMENT – 2
Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.
Sol. Since Statement–2 is false.
Key **(C)**

34. STATEMENT – 1
For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.
because
STATEMENT – 2
If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement– 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.
Sol Both the statements are true but statement–2 does not explain statement–1.
Key **(B)**

35. STATEMENT – 1
It is easier to pull a heavy object than to push it on a level ground.
because
STATEMENT – 2
The magnitude of frictional force depends on the nature of the two surfaces in contact.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is** a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement– 1.

(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol Both the statements are true but Statement–2 does not explain statement–1.

Key **(B)**

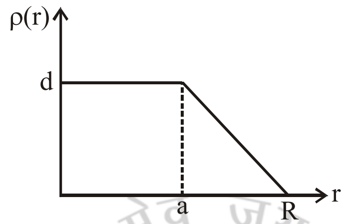
SECTION – III
Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 36 to 38

The nuclear charge (Ze) is non–uniformly distributed within a nucleus of radius R. the charge density ρ (r) [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.

Figure :



36. The electric field at r = R is
(A) independent of a
(B) directly proportional to a
(C) directly proportional to a^2
(D) inversely proportional to a.

Sol. For a spherically symmetric charge distribution, the electric field at outside points is independent of how the charge is distributed in the volume.

Key **(A)**

37. For a = 0, the value of d (maximum value of ρ as shown in the figure) is
(A) $\frac{3Ze}{4\pi R^3}$
(B) $\frac{3Ze}{\pi R^3}$
(C) $\frac{4Ze}{3\pi R^3}$
(D) $\frac{Ze}{3\pi R^3}$.

Sol. For a = 0

$$\rho = d - \frac{r}{R}d$$

Now $\int_0^R \left(d - \frac{r}{R}d\right) 4\pi r^2 dr = Ze \Rightarrow \quad d = \frac{3Ze}{\pi R^3}$

Key **(B)**

38. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies
(A) a = 0
(B) $a = \frac{R}{2}$
(C) a = R
(D) $a = \frac{2R}{3}$.

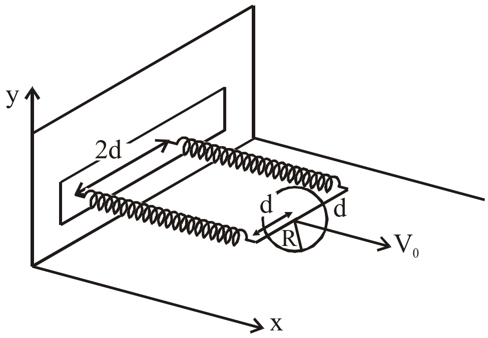
Sol. If ρ is constant then the $E \propto r$.

Key **(C)**

Paragraph for Question Nos. 39 to 41

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ.

Figure :

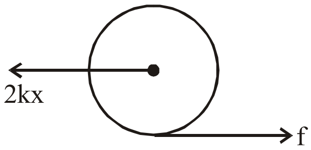


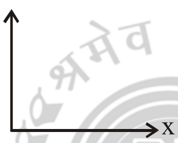
39. The net external force acting on the disc when its centre of mass is at displacement x with respect to its equilibrium position is

(A) $-kx$ (B) $-2kx$

(C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$.

Sol. F.D.B of the disc





$$\left.\begin{aligned} 2kx - f &= ma \\ fR &= \frac{1}{2}mR^2 \propto \\ \text{and } a &= R\propto \end{aligned}\right\} \Rightarrow f = \frac{2}{3}kx$$

$\therefore$ Net force $= -\frac{4}{3}kx$

Key **(D)**

40. The centre of mass of the disc undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ (B) (B) $\sqrt{\frac{2k}{M}}$

(C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$.

Sol $-\frac{4}{3}kx = ma$

$\Rightarrow \quad a = -\frac{4}{3}\frac{k}{m}x$

$\therefore \quad \omega = \sqrt{\frac{4k}{3m}}$

Key **(D)**

41. The maximum value of V_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$

(C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$.

Sol Now $\frac{2}{3}kA \le \mu Mg$ where A is the amplitude

Also $V_0 = A\omega \;\Rightarrow\; A = V_0\sqrt{\frac{3M}{4k}}$

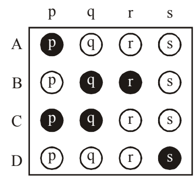
$$\therefore\; \frac{2}{3}k\, V_0\sqrt{\frac{3M}{4k}} \le \mu Mg$$

$$\Rightarrow\; V_0 \le \mu g\sqrt{\frac{3M}{k}}$$

Key **(C) is correct.**

SECTION – IV
Matrix Match Type

*This section contains 3 questions. Each question contains statements given in two column which have to be matched. Statements (A, B, C, D) in **Column I** have to be matched with statements (p, q, r, s) in **Column II**. The answers to these questions have to be appropriately bubbles as illustrated in the following example.*
If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows :



42. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I

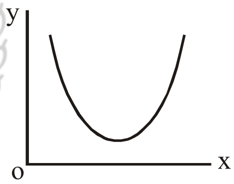
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)

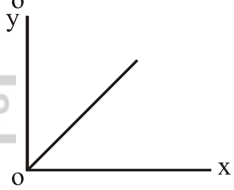
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x–direction.

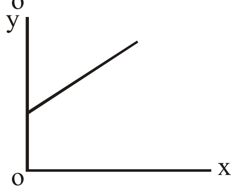
(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle

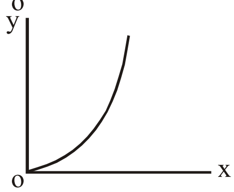
(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)

Column II

(p)


(q)


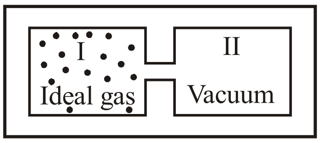
(r)


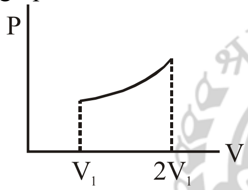
(s)


Key **(A) – (p)**
(B) – (q), (s)
(C) – (s)
(D) – (q)

43. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 given in the ORS.

Column I		**Column II**	
(A)	An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.	(p)	The temperature of the gas decreases
(B)	An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$ where V is the volume of the gas	(q)	The temperature of the gas increases or remains constant
(C)	An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$ where V is the volume of the gas	(r)	The gas loses heat
(D)	An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph	(s)	The gas gains heat

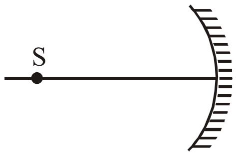




Key **(A) – (q)**
(B) – (p), (r)
(C) – (p), (s)
(D) – (q), (s)

44. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

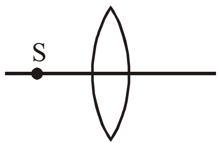
Column I | **Column II**

(A)


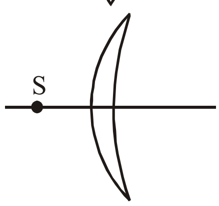
(p) Real image

(B)


(q) Virtual image

(C)


(r) Magnified image

(D)


(s) Image at infinity

Key. **(A) – (p), (q), (r), (s)**
(B) – (q)
(C) – (p), (q), (r), (s)
(D) – (p), (q), (r), (s)

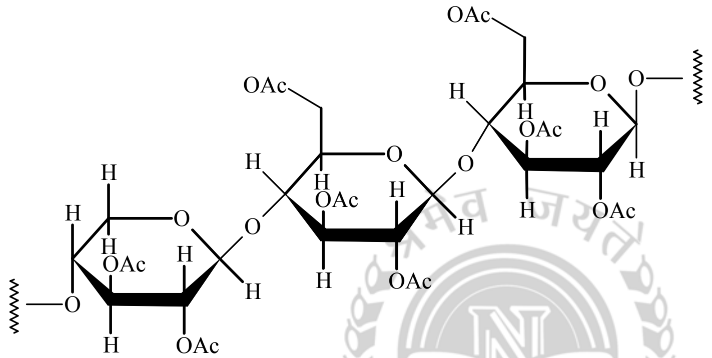
CHEMISTRY

SECTION – I
Straight Objective Type

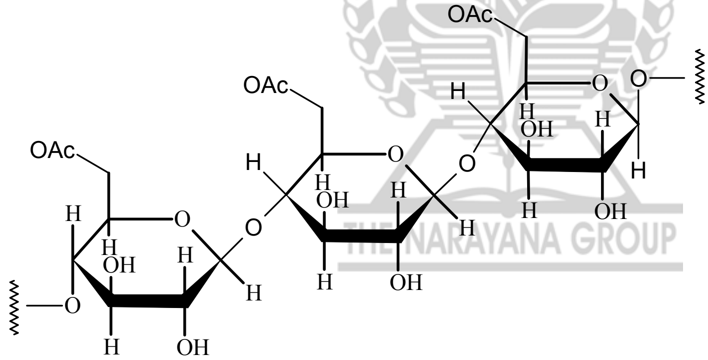
This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

45. Cellulose upon acetylation with excess acetic anhydride/H_2SO_4 (catalytic) gives cellulose triacetate whose structure is

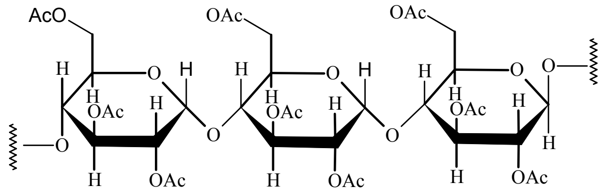
(A)



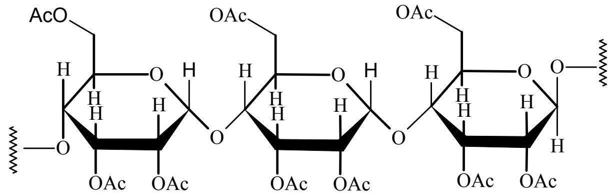
(B)



(C)



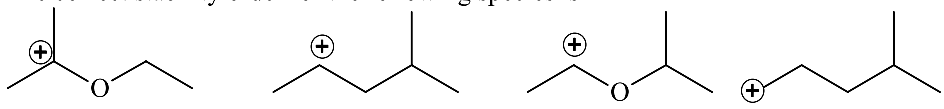
(D)



45. Cellulose in the straight chain polysaccharide composed of β-D-glucose units which are joined by glycosidic linkage between c-1 of one glucose unit and c-4 of the next glucose unit. Since it cellulosetriacelate so, its structure is as given in B.

Key **(B)**

*46. The correct stability order for the following species is



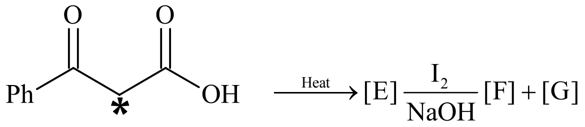
(I) (II) (III) (IV)

(A) (II) > (IV) > (I) > (III)
(B) (I) > (II) > (III) > (IV)
(C) (II) > (I) > (IV) > (III)
(D) (I) > (III) > (II) > (IV)

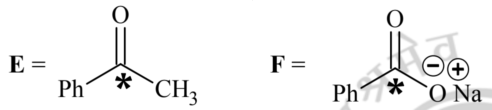
Sol The cabocation (I) is stabilized by six α-H atom as well as by +R effect of alkoxy group attached. III is stabilized by +R effect and by three α-H atoms. II is stabilished only by five α - H atoms while IV being 1° carbocation is stabilized by two α-H atoms.

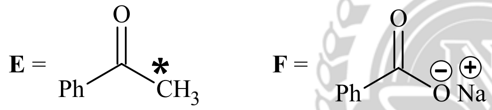
Key **(D)**

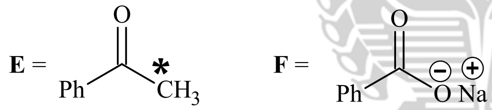
47. In the following reaction sequence, the correct structures of E, F and G are

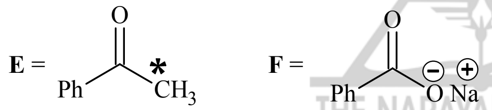


(* implies ^{13}C labelled carbon)

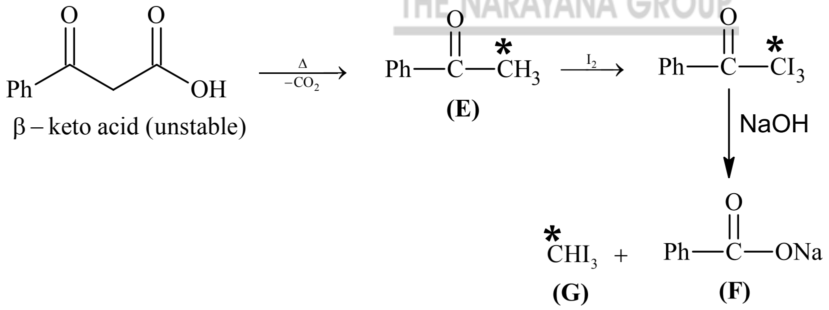
(A)  **G** = CHI_3

(B)  **G** = CHI_3

(C)  **G** = $\overset{*}{C}HI_3$

(D)  **G** = $\overset{*}{C}HI_3$

Sol.



Key **(C)**

48. Among the following, the surfactant that will form micelles in aqueous solution at the lowest molar concentration at ambient conditions is

(A) $CH_3(CH_2)_{15}N^+(CH_3)_3Br^-$
(B) $CH_3(CH_2)_{11}\ OSO_3^-Na^+$
(C) $CH_3(CH_2)_6COO^-Na^+$
(D) $CH_3(CH_2)_{11}N^+(CH_3)_3Br^-$

Key **(B)**

49. Electrolysis of dilute aqueous NaCl solution was carried out by passing 10 milli ampere current. The time required to liberate 0.01 mol of H_2 gas at the cathode is (1 Faraday = 96500 C mol^{-1})

(A) 9.65×10^4 sec
(B) 19.3×10^4 sec
(C) 28.95×10^4 sec
(D) 38.6×10^4 sec

Sol. Charge passed = $\frac{10\times10^{-3}\times t}{96500}F$

0.01 mole H_2 = 0.02 equi. hydrogen

$\frac{10\times10^{-3}\times t}{96500}=0.02$

$t=\frac{0.02\times96500}{10^{-2}}=19.3\times10^{4}s$

Key **(B)**

*50. Solubility product constants (K_{sp}) of salts of types MX, MX_2 and M_3X at temperature 'T' are 4.0×10^{-8}, 3.2×10^{-14} and 2.7×10^{-15}, respectively. Solubilities (mol dm^{-3}) of the salts at temperature 'T' are in the order

(A) $MX > MX_2 > M_3X$
(B) $M_3X > MX_2 > MX$
(C) $MX_2 > M_3X > MX$
(D) $MX > M_3X > MX_2$

Sol $K_{sp(MX)}=s^2 \quad \therefore \quad s=\sqrt{4\times10^{-8}}=2\times10^{-4}M$

$K_{sp(MX_2)}=(2s)^2(s)=4s^3 \quad \therefore \quad s=\sqrt[3]{\frac{3.2\times10^{-14}}{4}}=2\times10^{-5}M$

$K_{sp(M_3X)}=(3s)^3(s)=27s^4 \quad \therefore \quad s=\sqrt[4]{\frac{2.7\times10^{-15}}{27}}=10^{-4}M$

Hence $MX > M_3X > MX_2$

Key **(D)**

51. Among the following, the coloured compound is

(A) CuCl
(B) $K_3[Cu(CN)_4]$
(C) CuF_2
(D) $[Cu(CH_3CN)_4]BF_4$

Sol. Cu^{2+} as in CuF_2 with incomplete d-orbital (d^9- configurtation) will exhibit colour due to d-d transition. In all the other choices Cu exists as Cu^+ with $3d^{10}$ configuration and hence colourless.

Key **(C)**

52. The IUPAC name of $[Ni(NH_3)_4]\ [NiCl_4]$ is

(A) Tetrachloronickel (II) – tetraamminenickel (II)
(B) Tetraamminenickel (II) – tetrachloronickel (II)
(C) Tetraamminenickel (II) – tetrachloronickelate (II)
(D) Tetrachloronickel (II) – Tetraamminenickelate (0)

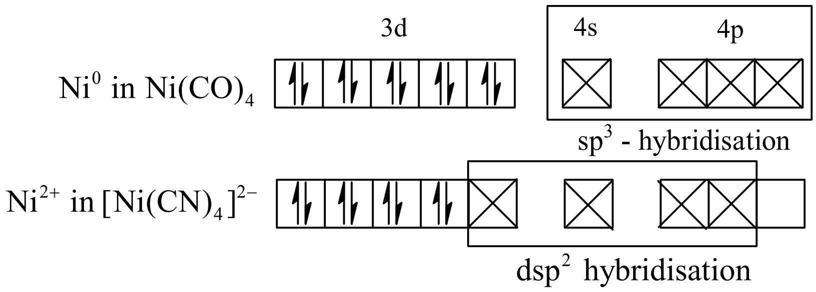
Sol. The cationic complex is named first followed by the name of anionic complex. The name of the metal in anionic complex ends with the suffix ate (See IUPAC nomenclature rules in detail).

Key **(C)**

53. Both $[Ni\ (CO)_4]$ and $[Ni\ (CN)_4]^{2-}$ are diamagnetic. The hybridizations of nickel in these complexes, respectively, are

(A) sp^3, sp^3
(B) sp^3, dsp^2
(C) dsp^2, sp^3
(D) dsp^2, dsp^2

Sol.



Note that both : CO and :CN^- are strong field ligands and are able to induce spin pairing.

Key (B)

SECTION – II
Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

54. STATEMENT-1
The geometrical isomers of the complex $[M(NH_3)_4Cl_2$ are optically inactive.
and
STATEMENT-2
Both geometrical isomers of the complex $[M\ (NH_3)_4Cl_2]$ possess axis of symmetry.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Key **(A)**

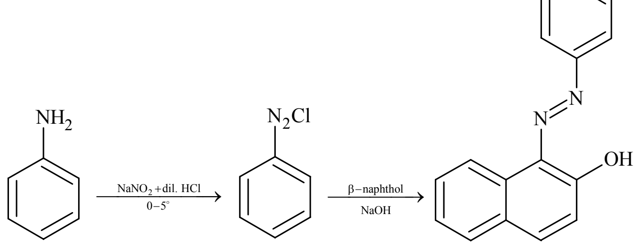
*55. STATEMENT-1
There is a natural asymmetry between converting work to heat and converting heat to work.
and
STATEMENT-2
No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol It is only in the cyclic process that the absorption of heat from a reservoir and its complete conversion into work is not possible and not in any process as stated.

Key **(C)**

56. STATEMENT-1
Aniline on reaction with $NaNO_2/HCl$ at 0°C followed by coupling with β - naphthol gives a dark blue coloured precipitate.
and
STATEMENT-2
The colour of the compound formed in the reaction of aniline with $NaNO_2/HCl$ at 0°C followed by coupling with β-naphthol is due to the extended conjugation.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol

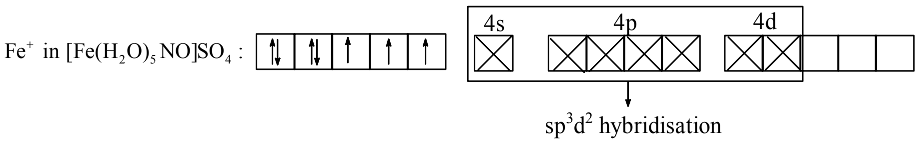


(brilliant red colour)
azo dye

Key **(D)**

57. STATEMENT-1
$[Fe(H_2O)_5NO]\ SO_4$ is paramagnetic.
and
STATEMENT-2
The Fe in $[Fe(H_2O)_5\ NO]SO_4$ has three unpaired electrons.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol In the complex $[Fe(H_2O)_5NO]SO_4$, the odd e^- on N-atom of NO molecule is transferred to the incomplete d-orbital of Fe^{2+} which changes Fe^{2+} into Fe^+ and NO into NO^+. H_2O and $\overset{+}{N}O$ both being weak field ligand fail to cause spin-pairing into Fe^+ as shown below:



As evident there are three unpaired electrons and hence paramagnetic

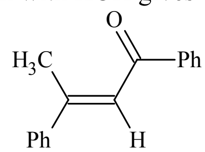
Key (A)

SECTION – III
Linked Comprehension Type

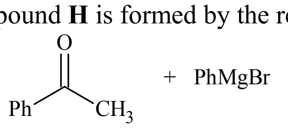
This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

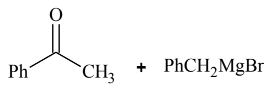
Paragraph for Questions Nos. 58 to 60

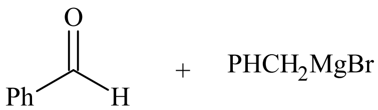
A tertiaryalcohol **H** upon acid catalysed dehydration gives a product **I**. Ozonolysis of I leads to compounds **J** and **K**. Compound **J** upon reaction with KOH gives benzyl alcohol and a compound **L**, whereas **K** on reaction with KOH gives only **M**.

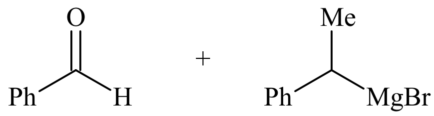
M = 

58. Compound **H** is formed by the reaction of

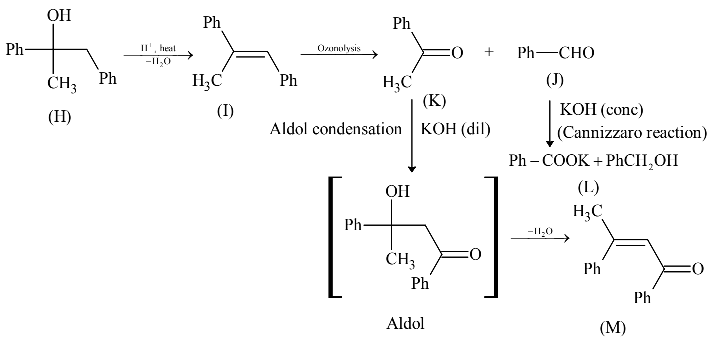
(A) 

(B) 

(C) 

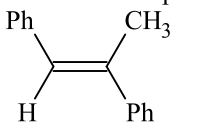
(D) 

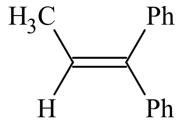
Sol

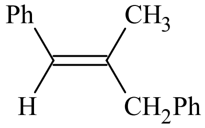


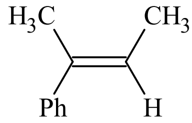
Key **(B)**

59. The structure of compound **I** is

(A) 

(B) 

(C) 

(D) 

Key **(A)**

60. The structures of compounds **J**, **K** and **L**, respectively, are

(A) $PhCOCH_3$, $PhCH_2COCH_3$ and $PhCH_2COO^-K^+$

(B) $PhCHO$, $PhCH_2CHO$ and $PhCOO^-K^+$

(C) $PhCOCH_3$, $PhCH_2CHO$ and $CH_3COO^-K^+$

(D) $PhCHO$, $PhCOCH_3$ and $PhCOO^-K^+$

Key **(D)**

Paragraph for Questions Nos. 61 to 63

In hexagonal systems of crystals, a frequently encountered arrangement of atoms is described as a hexagonal prism. Here, the top and bottom of the cell are regular hexagons and three atoms are sandwiched in between them. A space-filling model of this structure, called hexagonal close-packed (HCP), is constituted of a sphere on a flat surface surrounded in the same plane by six identical spheres as closely as possible. Three spheres are then placed over the first layer so that they touch each other and represent the second layer. Each one of thee three spheres touches three spheres of the bottom layer. Finally, the second layer is covered with a third layer that is identical to the bottom layer in relative position. Assume radius of every sphere to be 'r'.

61. The number of atoms in this HCP unit cell is

(A) 4 (B) 6

(C) 12 (D) 17

Sol Effective number of atoms present in unit cell of HCP

$$= 2\times\frac{1}{2}+3\times1+12\times\frac{1}{6}=6$$

Key **(B)**

62. The volume of this HCP unit cell is

(A) $24\sqrt{2}\,r^3$ (B) $16\sqrt{2}\,r^3$

(C) $12\sqrt{2}\,r^3$ (D) $\frac{64}{3\sqrt{3}}r^3$

Sol Volume = $6\times\frac{\sqrt{3}}{4}a^2\times h = 6\times\frac{\sqrt{3}}{4}a^2\times2\times\sqrt{\frac{2}{3}}a$

putting a = 2r

$$6\times\frac{\sqrt{3}}{4}(2r)^2\times2\sqrt{\frac{2}{3}}\cdot2r = 6\times\frac{\sqrt{3}}{4}\times4r^2\times2\sqrt{\frac{2}{3}}\cdot2r = 24\sqrt{2}r^3$$

Key **(A)**

63. The empty space in this HCP unit cell is

(A) 74% (B) 47.6%

(C) 32% (D) 26%

Sol Packing fraction in HCP = 0.74

$\therefore$ void fraction = 1 – 0.74 = 0.26

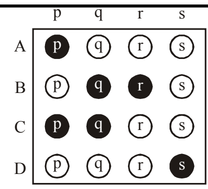
$\therefore$ empty space in HCP unit cell = 26%

Key **(D)**

SECTION – IV

Matrix Match Type

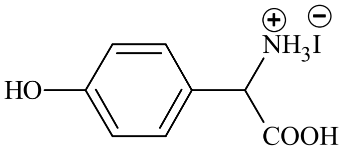
*This section contains 3 questions. Each question contains statements given in two column which have to be matched. Statements (A, B, C, D) in **Column I** have to be matched with statements (p, q, r, s) in **Column II**. The answers to these questions have to be appropriately bubbles as illustrated in the following example.*
If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows :

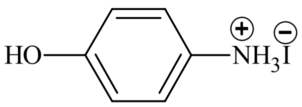


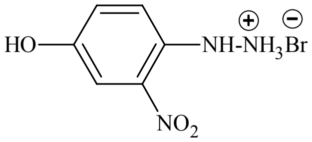
64. Match the entries in Column I with the correctly related quantum number(s) in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I

(A) $NH_2-\overset{\oplus}{N}H_3\overset{\ominus}{Cl}$

(B)


(C)


(D)


Column II

(p) sodium fusin extract of the compound gives Prussian blue colour with $FeSO_4$

(q) gives positive $FeCl_3$ test

(r) gives white precipitate with $AgNO_3$

(s) reacts with aldehyes to form the corresponding hydrazone derivative

Sol. (A) Though it contains N will nto give lassaigne test as there is no carbon present in it.
Although $FeCl_3$ has not been mentioned in p yet some Fe^{2+} ion getting oxidized by aerial oxides into Fe^{3+} is sufficient to form prussion blue.
Positive $FeCl_3$ test is given by phenolic —OH present with molecule. While ppt with $AgNO_3$ solution will be given by Cl^- ion.

Key **(A) – (r), (s)**
(B) – (p), (q)
(C) – (p), (q), (r)
(D) – (p)

*65. Match the entries in Column I with the correctly related quantum number(s) in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I

(A) Orbital angular momentum of the electron in a hydrogen-like atomic orbital.

(B) A hydrogen-like one-electron wave function obeying Pauli principle

(C) Shape, size and orientation of hydrogen like atomic orbitals

(D) Probability density of electron at the nucleus in hydrogen-like atom

Column II

(p) Principal quantum number

(q) Azimuthal quantum number

(r) Magnetic quantum number

(s) Electron spin quantum number

Sol Orbital angular momentum (L) = $\sqrt{\ell(\ell+1)}\dfrac{h}{2\pi}$

ℓ = azimuthal Q.N.
n determines size, ℓ shape and m – orientation of orbital.
The more function of H-like atom is spherical and has radial part only. It is independent of θ and ϕ, the angular parameters. Pauli principle concerns with the spin of electron it states that an orbital can contain only a maximum of two electrons and that two when their direction of spin are opposite.

Key **(A) – (q)**
(B) – (p), (q), (r), (s)
(C) – (p), (q), (r)
(D) – (p)

66. Match the entries in Column I with the correctly related quantum number(s) in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) $PbS \rightarrow PbO$	(p) roasting
(B) $CaCO_3 \rightarrow CaO$	(q) calcinations
(C) $ZnS \rightarrow Zn$	(r) carbon reduction
(D) $Cu_2S \rightarrow Cu$	(s) self reduction

Sol $2PbS + 3O_2 \xrightarrow{heat} 2PbO + 2SO_2 \uparrow$ (Roasting)

$CaCO_3 \xrightarrow{heat} CaO + CO_2 \uparrow$ (Calcination)

$2ZnS + 3O_2 \longrightarrow 2ZnO + 2SO_2$ (Roasting)

$ZnO + C \longrightarrow Zn + CO$ (Carbon – reduction)

$Cu_2S + \frac{3}{2}O_2 \longrightarrow Cu_2O + SO_2\uparrow$ (Roasting)

$Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2\uparrow$ (self-reduction)

$Cu_2O + C \longrightarrow 2Cu + CO$ (carbon – reduction)

Key **(A) – (p), (B) – (q), (C) – (p), (r), (D) – (p), (r), (s)**

MARKING SCHEME

PAPER – I

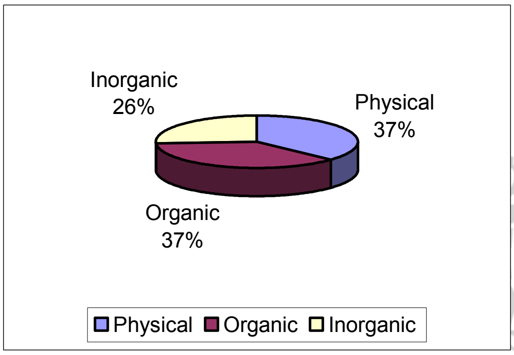
1. For each questions in **Section I**, you will be **awarded 3 marks** if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
2. For each question in **Section II**, you will be **awarded 4 marks** if you have darkened all the bubble(s) corresponding to the correct answer and zero mark for all other cases. If may be noted that there is **no negative marking for wrong answer.**
3. For each question in **Section III**, you will be **awarded 3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
4. For each question in **Section IV**, you will be **awarded 4 marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark will be awarded.**

PAPER – II

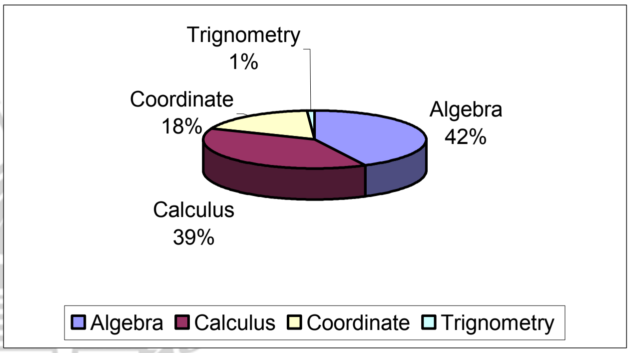
1. For each question in **Section – I**, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1)** mark will be awarded.
2. For each question in **Section II**, you will be **awarded 3 marks s** if you have darkened all the bubble(s) corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
3. For each question in **Section III**, you will be **awarded 4 marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
4. For each question in **Section IV**, you will be **awarded 6 marks** if you darken only the bubble corresponding only to the correct answer or awarded 1 mark each for correct bubbling of answer in any row. **No negative mark will be awarded for an incorrectly bubbled answer.**

WEIGHTAGE

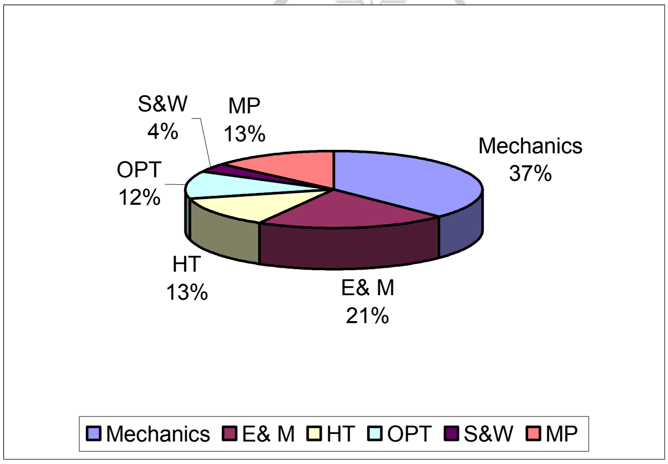
CHEMISTRY



MATHS



PHYSICS



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