IIT-JEE - 2008

IIT-JEE SOLUTIONS

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IIT-JEE 2008 SOLUTIONS

PAPER - I

Mathematics

1. If $0 < x < 1$, then $\sqrt{1+x^2}\ [\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$ =

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Sol.: $\sqrt{1+x^2}\left[\left\{\frac{x^2}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{1/2}$

$= \sqrt{1+x^2}\ \left[1+x^2-1\right]^{1/2} = x\ \sqrt{1+x^2}$.

Key: (C)

2. Consider the two curves

$C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 - 6x + 1 = 0$

Then,

(A) C_1 and C_2 touch each other only at one point

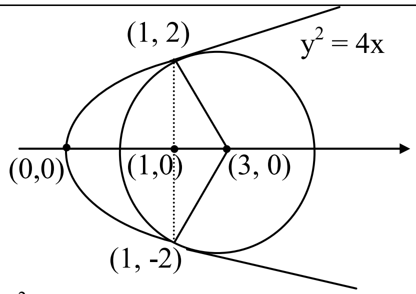
(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

Sol.:

$(x - 3)^2 + (y - 0)^2 = (2\ \sqrt{2}\ )^2$



$x^2 + 4x - 6x + \ = 0$

$(x - 1)^2 = 0$

x = 1, 1

Ans. touch each other exactly two points.

Key (B)

3. The edges of a parallelopiped are of unit length and are parallel to non–coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = \frac{1}{2}$

Then, the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol.:

$$V^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}$$

$\Rightarrow V^2 = \frac{1}{2}$

$\Rightarrow V = \frac{1}{\sqrt{2}}$

Key (A)

4. Let a and b be non–zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

Sol.:

$x^2 - 5xy + 6y^2 = 0$ represent a pair of lines passing through origin

$ax^2 + ay^2 = -c$

$\Rightarrow x^2 + y^2 = \frac{-c}{a} > 0$

$\Rightarrow ax^2 + ay^2 + c = 0$ represent a circle

Key (B)

5. The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is

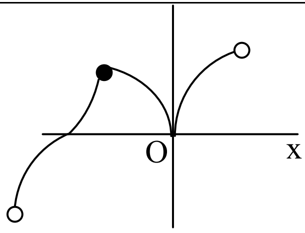
(A) 0 (B) 1

(C) 2 (D) 3

Sol.:

$f(x) = (2 + x)^3, \ -3 < x \le -1$

$\quad = x^{2/3}, \quad -1 < x < 2$



The total number of local maximum or minimum = 2.
Key (C)

6. Let $g(x) = \dfrac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand derivative of $|x - 1|$ at x = 1. If $\lim\limits_{x\to 1+} g(x) = p$, then

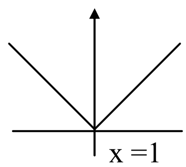
(A) n = 1, m = 1
(B) n = 1, m = –1
(C) n = 2, m = 2
(D) n > 2, m = n

Sol.: According to question we get,

$\lim\limits_{x\to 1^+} g(x) = -1$

$\Rightarrow \lim\limits_{h\to 0^+} \dfrac{(h)}{\log\cos^2(h)} = \lim\limits_{h\to 0^+} \dfrac{(h)^2}{2\log(\cosh)} = \lim\limits_{h\to 0} \dfrac{-2h}{2\tanh} = -1$

Alternate:
From graph : p = –1



$g(x) = \dfrac{(x-1)^n}{\log(\cos^m(x-1)}$ $0 < x < 2$, m, $\neq 0$ $n \in N$

$g(x) = \lim\limits_{x\to 1^+} \dfrac{(x-1)^n}{\log\cos^m(x-1)} = -1$

$g(1+) = \lim\limits_{h\to 0} \dfrac{(h)^n}{\log\cos^m h}$, $h > 0$

$= \lim\limits_{h\to 0} \dfrac{h^n}{m(\ln\cosh)}$

$= \dfrac{1}{m}\lim\limits_{h\to 0} \dfrac{h^n}{(\ln\cosh)}$

$= \dfrac{1}{m}\lim\limits_{h\to 0} \dfrac{n\,h^{n-1}}{-\sinh} \times \cosh$

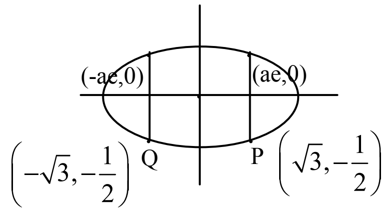
$= \dfrac{-n}{m}\lim\limits_{h\to 0} \dfrac{h^{n-2}}{\left(\dfrac{\sinh}{h}\right)} \times (\cosh)$

$\Rightarrow$ n = 2, m = 2
Key (C)

7. Let P (x_1, y_1) and Q(x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2 + 2\sqrt{3}\ y = 3 + \sqrt{3}$
(B) $x^2 - 2\sqrt{3}\ y = 3 + \sqrt{3}$
(C) $x^2 + 2\sqrt{3}\ y = 3 - \sqrt{3}$
(D) $x^2 - 2\sqrt{3}\ y = 3 - \sqrt{3}$

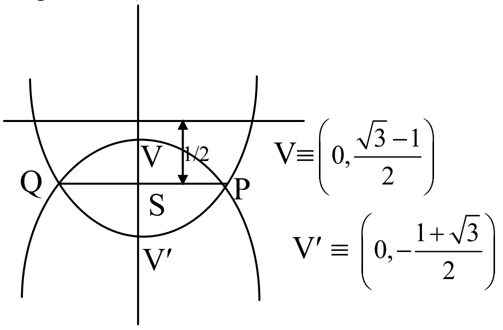
Sol.: Given ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1 \Rightarrow e = \sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$



$P \equiv (ae, -b^2/a) = \left(\sqrt{3},-\frac{1}{2}\right)$

$Q \equiv (-ae, -b^2/a) = (-\sqrt{3}, -\frac{1}{2})$

length of PQ = $2\sqrt{3}$



$VS = SV' = \frac{PQ}{4} = \frac{\sqrt{3}}{2}$

$\therefore$ Equations of parabolas are

$x^2 = -2\sqrt{3}\left(y-\frac{\sqrt{3}-1}{2}\right) \Rightarrow x^2 + 2\sqrt{3}\ y = 3 - \sqrt{3}$

and $x^2 = 2\sqrt{3}\left(y+\frac{1+\sqrt{3}}{2}\right) \Rightarrow x^2 - 2\sqrt{3}\ y = 3 + \sqrt{3}$

Key (B,C)

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

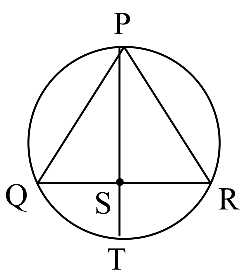
(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$

(B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$

(D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Sol.:



H.M. < G.M.

$\frac{2}{\frac{1}{PS}+\frac{1}{ST}} < (PS.\ ST)^{1/2}$

$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{PS.ST}}$

As PS × ST = QS × SR

$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$ ... (i) Option (B) is correct

A.M. > G.M

$\frac{QS+SR}{2}>\sqrt{(QS.SR)}$

$\therefore \sqrt{QS\times SR} < \frac{QR}{2}$ ... (ii)

From (i) and (ii)

$\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Key (D)

9. Let f(x) be a non–constant twice differentiable function defined on (–∞, ∞) such that f(x) = f(1–x) and $f'\left(\frac{1}{4}\right)$ = 0. Then,

(A) $f''(x)$ vanishes at least twice on [0, 1]

(B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right) \sin x \, dx = 0$

(D) $\int_{0}^{1/2} f(t)e^{\sin \pi t}\, dt = \int_{1/2}^{1} f(1-t)\, e^{\sin\pi t}\, dt$

Sol.: (A) $f'(x) = -f'(1 - x)$

$f'(1/4) = -f'(3/4) = 0$

$f'(3/4) = 0$

$f'(1/4) = 0$

$\Rightarrow \exists$ a point $\in$ (1/4, 3/4) in which $f''(x) = 0$

$f''(1/2) = 0$

(B) f(x) = f(1–x)

$f'(x) = -f'(1-x)$

$f'(1/2) = -f'(1/2)$

$f'(1/2) = 0.$

(C) f(x + 1/2) = f(1 – x – 1/2)

$\Rightarrow$ f(x + 1/2) = f(1/2 – x)

i.e., f(x + 1/2) is even

i.e., $\int_{-1/2}^{1/2} f(x+1/2)\sin x = 0$

(D) $\int_{0}^{1/2} f(t)\, e^{\sin\pi t} = \int_{1/2}^{1} f(1-t)\, e^{\sin\pi t}\, dt$

Put (1 – t) = u

$\Rightarrow$ R.H.S. $= -\int_{1/2}^{0} f(4)\, e^{\sin\pi(1-u)}\, du = \int_{0}^{1/2} f(u)\, e^{\sin\pi u}\, dx$

K ey (A,B,C,D)

10. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$ for n = 1, 2, 3, ... then ,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$

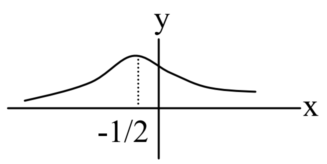
(B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$

(D) $T_n > \frac{\pi}{3\sqrt{3}}$

Sol.: Consider: $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{k^2+kn+n^2} = \int_0^1 \frac{dx}{x^2+x+1} = \frac{\pi}{3\sqrt{3}}$

$f(x) = \frac{1}{x^2+x+1} = \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$



As f(x) is decreasing for x > 0, $S_n < \int_0^1 \frac{1}{x^2+x+1}dx < T_n$

Hence A & D are correct.

Key: (A), (D)

SECTION – III

Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

11. Consider the system of equations

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1$$

STATEMENT–1: The system of equations has no solutions for $k \neq 3$

and

STATEMENT–2: The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1

(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1

(C) STATEMENT–1 is True, STATEMENT–2 is False

(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol.: $\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$

$\Delta_z = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & k \\ 1 & -3 & 1 \end{vmatrix} = 3 - k \neq 0, k \neq 3$

Key (A)

12. Consider the system of equations ax + by = 0, cx + dy = 0, where a, b, c, d $\in$ {0, 1}

STATEMENT–1: The probability that the system of equations has a unique solution is 3/8

and

STATEMENT–2: The probability that the system of equations has a solution is 1.

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1

(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1

(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol.: a, b, c, d $\in$ {0, 1}

$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $n(s) = 2^4 = 16$

(I) To have unique solution $\Delta \neq 0$.

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0, & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$ Total = 6 cases

The probability that system of equations has unique solution = 6/16 = 3/8

II. Homogenous system is always consistent

Key (B)

13. Let f and g be real valued functions defined on interval (–1, 1) such that $g''(x)$ is continuous $g(0) \neq 0$, $g'(0) = 0$ $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

STATEMENT–1: $\lim_{x \to 0} [g(x) \cot x - g(x) \operatorname{cosec} x] = f''(0)$.

and

STATEMENT–2: $f'(0) = g(0)$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol.: $f(x) = g(x) \sin x$

$f''(x) = g''(x) \sin x + 2g'(x) \cos x + g(x) \sin x$

$f''(0) = 2g'(0) = 0$

$f'(x) = g'(x) \sin x + g(x) \cos x$

$f'(0) = g(0)$

For statement 1: $\lim_{x \to 0} \frac{g(x)\cos x - g(0)}{\sin x} = \lim_{x \to 0} \frac{g'(x)\cos x - g(x)\sin x}{\cos x} = g'(0) = 0 = f''(0)$

For statement 2

$f'(0) = g(0)$

Key: (B)

14. Consider three planes $P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively

STATEMENT–1: At least two of the lines L_1, L_2 and L_3 are non-parallel

and

STATEMENT–2: The three planes do not have a common point

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol.: L_1, L_2, L_3 are parallel to each other

$\Rightarrow$ statement (1) is not true

$$D = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ The three planes do not have a common point.
$\Rightarrow$ statement (2) is correct.
Key : (D)

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 15 to 17

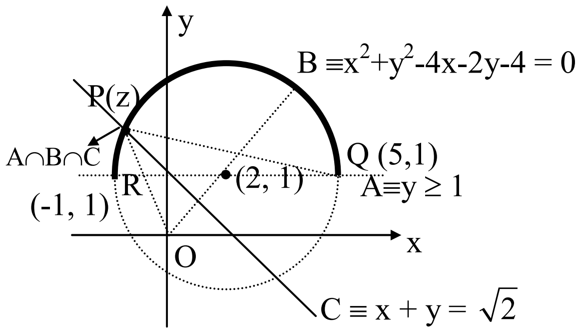
Let A, B, C be three sets of complex numbers as defined below
$A = \{z : \text{Im}z \geq 1\}$
$B = \{z : |z - 2 - i| = 3\}$
$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$

15. The number of element in the set $A \cap B \cap C$ is
(A) 0 (B) 1
(C) 2 (D) ∞

16. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between
(A) 25 and 29 (B) 30 and 34
(C) 35 and 39 (D) 40 and 44

17. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between
(A) –6 and 3 (B) –3 and 6
(C) –6 and 6 (D) –3 and 9

Sol (15 – 17)



15. From graph only one point in $A \cap B \cap C$
Key (B)

16. (C)
$|z + 1 - i|^2 + |z - 5 - i|^2$
$= PR^2 + PQ^2 = RQ^2 = 6^2 = 36$

17. (D)
As $||z| - |\omega|| < |z - \omega| < 6$
$\Rightarrow ||z| - |\omega|| < 6$
$\Rightarrow -6 < |z| - |\omega| < 6$ or $-6 + 3 < |z| - |\omega| + 3 < 9$
$\Rightarrow -3 < |z| - |\omega| + 3 < 9$
Key (D).

Paragraph for Questions Nos. 18 to 20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D,E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

(A) $(x-2\sqrt{3})^2 + (y-1)^2 = 1$ (B) $(x-2\sqrt{3})^2 + (y+\frac{1}{2})^2 = 1$

(C) $(x-\sqrt{3})^2 + (y+1)^2 = 1$ (D) $(x-\sqrt{3})^2 + (y-1)^2 = 1$

19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

20. Equations of the sides QR, RP are

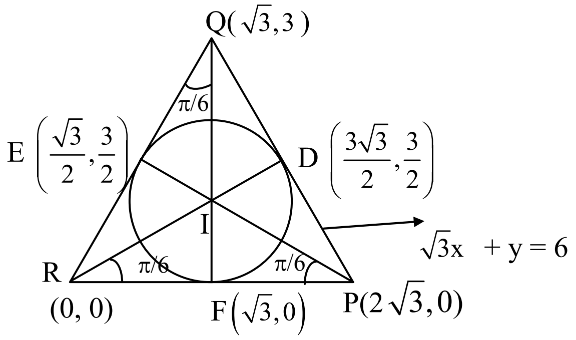
(A) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x,\ y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1,\ y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x,\ y = 0$

Sol.: 18–20

$P \equiv \left(\frac{3\sqrt{3}}{2} - \sqrt{3}\times\left(-\frac{1}{2}\right), \frac{3}{2} - \sqrt{3}\times\frac{\sqrt{3}}{2}\right)$

$= \left(2\sqrt{3}, 0\right)$



I = ($\sqrt{3}$, 1)

18. $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Key : (D)

19. E = $\sqrt{3}\cos\frac{\pi}{3}, \sqrt{3}\sin\frac{\pi}{3} \equiv \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

F ≡ ($\sqrt{3}$, 0)

Key : (A)

20. Equation of PR, y = 0

Equation of QR, y = $\sqrt{3}$ x

Key (D)

Paragraph for Questions Nos. 21 to 23

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.
If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function y = f(x).

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

21. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$

(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

22. The area of the region bounded by the curve $y = f(x)$, the x–axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3[((f(x))^2 - 1]} dx + bf(b) - af(a)$ (B) $-\int_a^b \frac{x}{3((f(x))^2 - 1)} dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$ (D) $-\int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$

23. $\int_{-1}^{1} g'(x)\, dx =$

(A) $2g(-1)$ (B) 0

(C) $-2g(1)$ (D) $2g(1)$

Sol.: 21 – 23

$y^3 - 3y + x = 0$

$\Rightarrow 3y^2 y' - 3y' + 1 = 0 \Rightarrow y' = \frac{1}{3(1-y^2)}$

$\Rightarrow y'' = \frac{6y(y')^2}{3(1-y^2)} = \frac{2y}{(1-y^2)^3.9} = \frac{2.2\sqrt{2}}{(1-8)^3.9} = -\frac{4\sqrt{2}}{7^3 3^2}$

Key (B)

22. $x = -y^3 + 3y$

as $x \in (-\infty, -2) \Rightarrow x < -2$

$\Rightarrow -y^3 + 3y < -2$

$\Rightarrow y^3 - 3y - 2 > 0$

$\Rightarrow (y + 1)^2 (y - 2) > 0$

$\Rightarrow y > 2\ \forall\ x \in (-\infty, -2)$ (As $y = -1 \Rightarrow x = -2$)

$\Rightarrow f(x)$ is positive $\forall x \in (-\infty, -2)$

Hence required area = $\int_a^b f(x)\,dx = \int_a^b y \cdot 1dx = yx\big|_a^b - \int_a^b \frac{dy}{dx} xdx$

$= \int_a^b \frac{xdx}{3((f(x))^2 - 1)} + bf(b) - af(a)$

Key (A)

23. Consider: $(g(x))^3 - 3g(x) + x = 0$

and $(g(-x))^3 - 3g(-x) - x = 0$

$\Rightarrow (g(x))^3 + (g(-x))^3 - 3(g(x) + g(-x)) = 0$

$\Rightarrow [g(x) + g(-x)]\,[(g(x))^2 + (g(-x))^2 - g(x)\,g(-x) - 3] = 0$

Let $(g(x))^2 + (g(-x))^2 - g(x)\,g(-x) - 3 = 0$

$\Rightarrow g(0)^2 = 3$

$\Rightarrow g(0) = +\sqrt{3}$ or $-\sqrt{3}$ which is not the case as $g(0) = 0$, given

$\Rightarrow g(x) + g(-x) = 0 \Rightarrow g(x)$ is an odd function for $x \in (-2, 2)$

$\Rightarrow \int_{-1}^{1} g'(x)\, dx = g(1) - g(-1) = 2g(1).$

Key (D)

P h y s i c s

Useful Data :	
Planck's constant	$h = 4.1 \times 10^{-14}$ eV.s
Velocity of light	$c = 3 \times 10^{8}$ m/s.

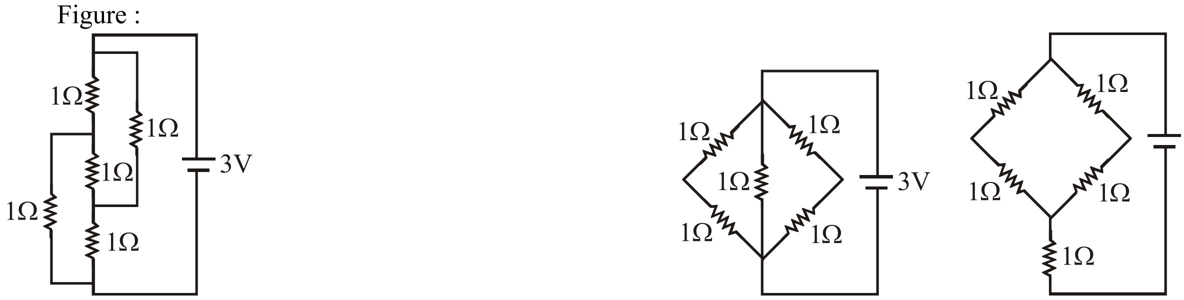
SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

24. Figure shows three resistor configurations R1, R2 and R3 connected to 3V battery. If the power dissipated by the configuration R1, R2 and R3 is P1, P2 and P3, respectively, then

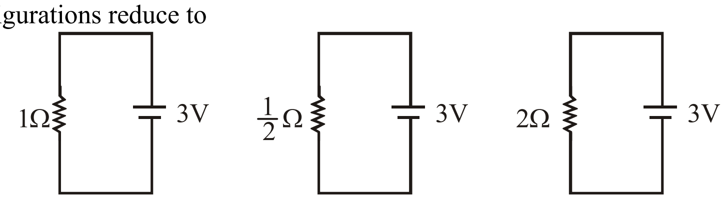
Figure :



R1 R2 R3

(A) P1 > P2 > P3
(B) P1 > P3 > P2
(B) P2 > P1 > P3
(D) P3 > P2 > P1.

Sol. The resistor configurations reduce to



R1 R2 R3

Now $P = \frac{V^2}{R_{eq}}$

$\therefore$ P2 > P1 > P3

Key $\therefore$ (C) is correct.

25. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s.

Student	Length of the pendulum (cm)	Number of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students I, II and III, respectively,

(A) $E_I = 0$
(B) E_I is minimum
(C) $E_I = E_{II}$
(D) E_{II} is maximum.

Sol. $g = 4\pi^2 \frac{\ell}{T^2}$

$E = \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$

Student	Length of the pendulum, error (cm), ℓ, $\Delta\ell$	No. of Oscillation	Time Period, error T, ΔT
I	64.0±0.1	8	$16.0 \pm \frac{0.1}{8}$
II	64.0±0.1	4	$16.0 \pm \frac{0.1}{4}$
III	20.0±0.1	4	$9.0 \pm \frac{0.1}{4}$

$E_I = \frac{0.2}{64}$, $E_{II} = \frac{0.3}{64}$, $E_{III} = \frac{0.1}{20} + \frac{0.1}{18}$.

Key ∴ E_1 is the minimum correct choice is (B)

26. Which one of the following statement is WRONG in the context of X–rays generated from X–ray tube ?
(A) wavelength of characteristic X–rays decreases when the atomic number of the target increases
(B) cut–off wavelength of the continuous X–rays depends on the atomic number of the target
(C) intensity of the characteristic X–rays depends on the electrical power given to the X–ray tube
(D) cut–off wavelength of the continuous X–rays depends on the energy of the electrons in the X–ray tube.

Sol. Cut–off wavelength depends upon accelerating potential.
∴ (B) is the correct choice.

27. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60º). In the position of minimum deviation, the angle of refraction will be
(A) 30º for both the colours
(B) greater for the violet colour
(C) greater for the red colour
(D) equal but not 30º for both the colours.

Sol. In the position of minimum deviation

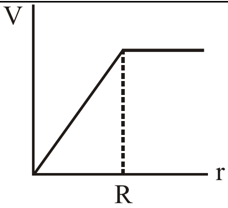
$r = \frac{A}{2}$ irrespective of colour

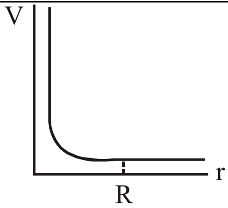
∴ (A) is correct.

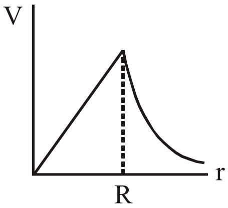
28. A spherically symmetric gravitational system of particles has a mass density

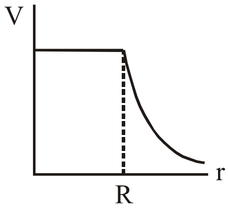
$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

Sol. For points $r \le R$,

$$\frac{G\left(\rho_0 \frac{4}{3}\pi r^3\right)m}{r^2} = \frac{mv^2}{r}$$

$\Rightarrow \quad v \propto r$

for points $r > R$

$$\frac{G\left(\rho_0 \frac{4}{3}\pi R^3\right)m}{r^2} = \frac{mv^2}{r}$$

$\Rightarrow \quad v \propto \frac{1}{\sqrt{r}}$

$\therefore$ (C) is correct.

29. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$

(C) $\frac{3}{T}$ (D) $\frac{4}{T}$.

Sol. Coefficient of volume expansion $\gamma = \frac{1}{V}\frac{dV}{dT}$

Now, $PT^2 = $ constant

$\Rightarrow \quad \frac{T^3}{V} = $ constant

$\therefore \quad \frac{3T^2.V - T^3\frac{dV}{dT}}{V^2} = 0$

$\Rightarrow \quad \gamma = \frac{3}{T}$.

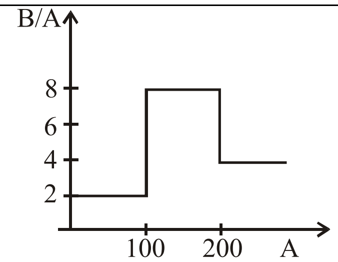
$\therefore$ (C) is correct.

SECTION II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

30. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice (s) given below.
Figure



(A) fusion of two nuclei with mass numbers lying in the range of 1 < A < 50 will release energy
(B) fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy
(C) fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments
(D) fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

Sol. (Energy released) = Total B.E. of products – Total B.E. of reactants.
$\therefore$ (B) and (D) are correct.

31. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is not external force acting on the balls. Let $\vec{p}_1^{'}$ and $\vec{p}_2^{'}$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non–zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{p}_1^{'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2^{'} = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{p}_1^{'} = c_1\hat{k}$, $\vec{p}_2^{'} = c_2\hat{k}$

(C) $\vec{p}_1^{'} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2^{'} = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}_1^{'} = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2^{'} = a_2\hat{i} + b_1\hat{j}$.

Sol. Initial momentum of the system $= \vec{0}$.
In the absence of external forces, final momentum of the system must also be zero.
i.e., $\vec{P}_1^{1} + \vec{P}_2^{1} = \vec{0}$
$\therefore$ (A) and (D) are correct.

32. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s),
(A) if $d = \lambda$, the screen will contain only one maximum
(B) if $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
(C) if the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(D) if the intensity of light falling on slit 2 is reduced so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase.

Sol. Condition for maxima is
$d \sin\theta = n\lambda$
If $d = \lambda$, $\sin\theta = n$
Possible value is 0
$\therefore$ only one maxima will be obtained
$\therefore$ (A) is correct.
IF $\lambda < d < 2\lambda$
$\Rightarrow \lambda < \dfrac{n\lambda}{\sin\theta} < 2\lambda$
$\Rightarrow 1 < \dfrac{n}{\sin\theta} < 2$
$\Rightarrow n = 0, \pm 1.$
$\therefore$ (B) is correct.

$I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$ & $I_{min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2$

with $I_1 = 4I_2$

$I_{max} = 9I_2,\ I_{min} = I_2$

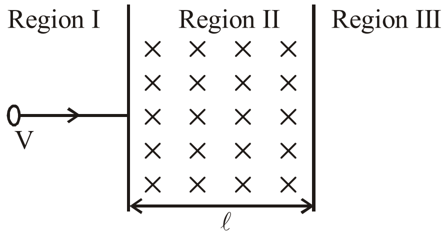
but when $I_1 = I_2$

$I_{max} = 4I_2$ and $I_{min} = 0$

$\therefore$ (C) and (D) are wrong.

33. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s)

Figure :



(A) the particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

(B) the particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) time spent in Region II is same for any velocity V as long as the particle returns to Region I.

Sol. The radius of the circular path in region II $r = \frac{mV}{qB}$

If $r > \ell$, the particle enters the region III

$\Rightarrow V > \frac{qB\ell}{m}$

$\therefore$ (A) is correct.

The path length will be maximum if it is able to describe a semi–circle

i.e., $\ell = r \Rightarrow V = \frac{q\ell B}{m}$

$\therefore$ (C) is correct.

If it is to return to region I, it must trace a semi–circle, the time for which is dependent of V $\left(T = \frac{\pi m}{qB}\right)$

$\therefore$ (D) is correct.

SECTION III

Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

34. STATEMENT – 1

Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

because

STATEMENT – 2

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol. The acceleration of an object down an incline of angle θ is

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

Now, $I_{hollow} > I_{solid}$ for same mass and dimensions
∴ $a_{hollow} < a_{solid}$
∴ solid cylinder will reach the bottom first.
∴ Statement–1 is false.
∴ (D) is correct.

35. STATEMENT – 1
The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.
because
STATEMENT – 2
In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol. As the water stream moves up, its speed decreases (due to gravity) and since flow rate (= Av) remains constant, the area increases making it spread like a fountain. The reverse is true when it moves down.
Both Statements are correct and Statement–2 is a correct explanation of Statement–1.
∴ (A) is correct.

36. STATEMENT – 1
In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.
because
STATEMENT – 2
Resistance of a metal increases with increase in temperature.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol. In a meter bridge

$$\frac{R_X}{R_S} = \frac{X}{100-X} \text{ at null point}$$

When unknown resistance is put in an enclosure maintained at a higher temperature, R_X will increase. To keep the null point same, R_S has to be increased.
∴ Statement–1 is false.
∴ (D) is correct.

37. STATEMENT – 1
An astronaut in an orbiting space station above the Earth experiences weightlessness.
because
STATEMENT – 2
An object moving around the Earth under the influence of Earth's gravitational force is in a state of free–fall.
(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1.
(C) Statement – 1 is True, Statement – 2 is False.
(D) Statement – 1 is False, Statement – 2 is True.

Sol. Statement–1 is correct and so is Statement–2. Both the astronaut and space station are in a state of free fall.
$\therefore$ (A) is correct.

Section IV

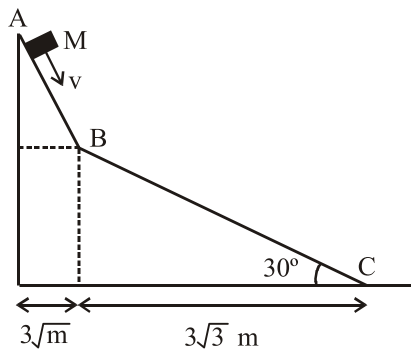
Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 38 to 40

A small block of mass M moves on a frictionless surface of an inclined plane, as show in figure. The angle of the incline suddenly changes from 60º to 30º at point B. The block is initially at rest at A. Assume that collision between the block and the incline are totally inelastic ($g = 10\ m/s^2$).

Figure :



38. The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}\ m/s$ (B) $\sqrt{45}\ m/s$

(C) $\sqrt{30}\ m/s$ (D) $\sqrt{15}\ m/s$.

Sol. At B before collision $v_0 = \sqrt{60}\ m/s$ at an angle of 30º with downward vertical.

Hence component of v_0 along BC = $v_0 \cos 30º$

$\therefore$ impact force acts along the normal to the plane, hence $v_0 \cos 30º$ will not change and component of v_0 perpendicular to the plane becomes zero as collision is completely inelastic.

$\therefore$ (B) is correct.

39. The speed of the block at point C immediately before it leaves the second incline is

(A) $\sqrt{120}\ m/s$ (B) $\sqrt{105}\ m/s$

(C) $\sqrt{90}\ m/s$ (D) $\sqrt{75}\ m/s$

Sol. Potential Energy + Kinetic Energy = constant

$\Rightarrow \frac{1}{2}m(v_0 \cos 30°)^2 + mg \times 3m = \frac{1}{2}mv_e^2$

$\Rightarrow v_c = \sqrt{105}\ m/s.$

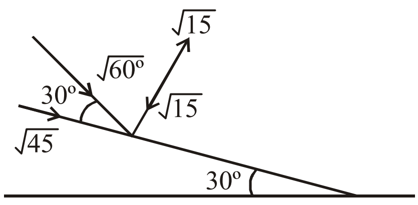
$\therefore$ (B) is the correct choice.

40. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

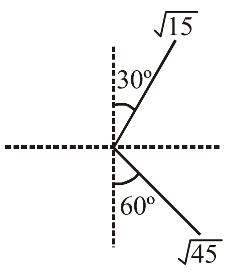
(A) $\sqrt{30}\ m/s$ (B) $\sqrt{15}\ m/s$

(C) 0 (D) $-\sqrt{15}\ m/s$.

Sol. Velocity after collision



$U_y = \sqrt{15}\cos 30° - \sqrt{45}\cos 60°$



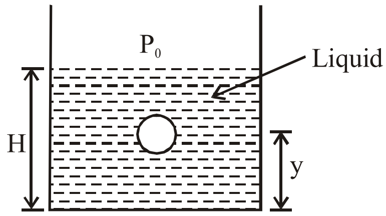
$= \frac{\sqrt{45}}{2} - \frac{\sqrt{45}}{2} = 0$.

$\therefore$ (C) is correct.

Paragraph for Question Nos. 41 to 43

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with eth liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure :



41. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(A) only the force of gravity

(B) the force due to gravity and the force due to the pressure of the liquid

(C) the force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(D) the force due to gravity and the force due to viscosity of the liquid.

Sol. Besides the buoyancy force, the other forces are

(i) force of gravity, and

(ii) force due to viscosity

The buoyant force is due to the pressure of the liquid.

$\therefore$ (D) is correct.

42. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{\frac{2}{5}}$ (B) $T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{\frac{2}{5}}$

(C) $T_0\left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{\frac{3}{5}}$ (D) $T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{\frac{3}{5}}$.

Sol. For the gas inside the bubble, the adiabatic equation $T^Y P^{1-Y}$ = constant applies

$\Rightarrow T_0^{\frac{5}{3}}(P_0 + \rho_\ell gH)^{-\frac{2}{3}} = T^{\frac{5}{3}}\{P_0 + \rho_\ell g(H-y)\}^{-\frac{2}{3}}$

$\Rightarrow T = \frac{T_0\{P_0 + \rho_\ell g(H-y)\}^{\frac{2}{5}}}{(P_0 + \rho_\ell gH)^{\frac{2}{5}}}$

$\therefore$ (B) is correct.

43. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0 \frac{(P_0 + \rho_\ell gH)^{\frac{2}{5}}}{(P_0 + \rho_\ell gy)^{\frac{7}{5}}}$ (B) $\frac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell gH)^{\frac{2}{5}}]P_0 + \rho_\ell g(H-y)]^{\frac{3}{5}}}$

(C) $\rho_\ell nRgT_0 \dfrac{(P_0+\rho_\ell gH)^{\frac{3}{5}}}{(P_0+\rho_\ell gy)^{\frac{8}{5}}}$

(D) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{\frac{3}{5}}[P_0+\rho_\ell g(H-y)]^{\frac{2}{5}}}$.

Sol. Buoyancy force = $V\rho_\ell g$

where V is the volume of the bubble (at location y)

Now $V_0 = \dfrac{nRT_0}{(P_0+\rho_\ell gH)}$

and PV^Y = constant

$$\Rightarrow \quad (P_0+\rho_\ell gH)\left\{\frac{nRT_0}{P_0+\rho_\ell gH}\right\}^Y = \left\{P_0+\rho_\ell(H-y)\right\}V^\gamma$$

$$\Rightarrow \quad V = nRT_0\frac{(P_0+\rho_\ell H)^{\frac{1-\gamma}{\gamma}}}{\left\{P_0+\rho_\ell g(H-y)\right\}^{\frac{1}{\gamma}}}$$

$\therefore$ Buoyant force = $V\rho_\ell g$

$\therefore$ (B) is correct.

Paragraph for Question Nos. 44 to 46

In a mixture of H – He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

44. The quantum number n of the state finally populated in He^+ ions is

(A) 2
(B) 3
(C) 4
(D) 5.

Sol. The excitation energy of the H atoms

= – 3.4 – (– 13.6) = 10.2 eV

Energy of the He^+ ions in their first excited state

= – 3.4 × 4 = – 13.6 eV

Energy after transferrence of energy

= – 13.6 + 10.2 = – 3.4 eV.

Now let n be the quantum number of the final state of He^+ ions, then

$$-3.4 = -13.6\frac{(2)^2}{n^2}$$

$\Rightarrow$ n = 4.

$\therefore$ (C) is correct..

45. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is

(A) 6.5×10^{-7} m
(B) 5.6×10^{-7} m
(C) 4.8×10^{-7} m
(D) 4.0×10^{-7} m.

Sol.

$$\Delta E = E_3 - E_4$$

$$\Rightarrow \quad \frac{hc}{\lambda} = E_0 \times 4 \times \frac{7}{144}$$

$$\lambda = \frac{12.4\times10^{-7}\,\text{eVm}\times36}{7\times13.6\,\text{eV}} \approx 4.8\times10^{-7}\,\text{m}$$

$\therefore$ (C) is correct.

46. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is

(A) $\dfrac{1}{4}$
(B) $\dfrac{1}{2}$

(C) 1 (D) 2.

Sol. Kinetic energy of n = 2 electron for H atom = 3.4 eV

Kinetic energy of n = 2 electron for He^+ ion = 3.4 × 4 eV

$\therefore$ ratio $= \frac{1}{4}$

$\therefore$ (A) is correct.

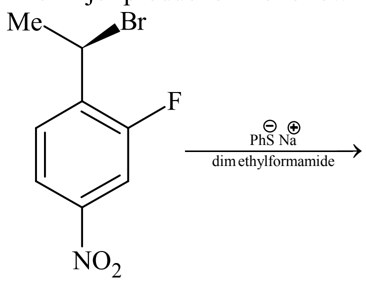
C h e m i s t r y

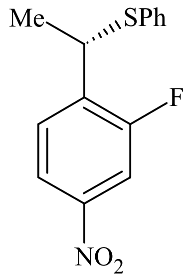
SECTION – I

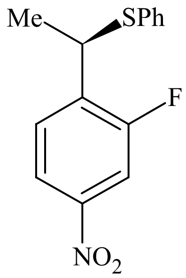
Straight Objective Type

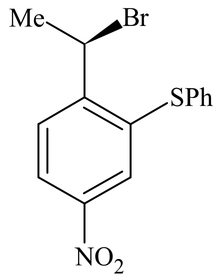
This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

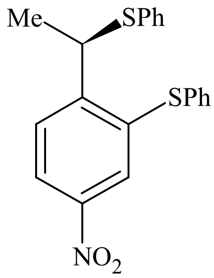
47. The major product of the following reaction is



(A) 

(B) 

(C) 

(D) 

Sol. In DMF solvent S_N1 being not possible, S_N2 takes place resulting into inversion of configuration.

Key **(A)**

48. Hperconjugation involves overlap of the following orbitals

(A) σ - σ (B) σ - p

(C) p – p (D) π - π

Sol Hyperconjuation is σ - π, σ - odd e^- and σ - cationic carbon conjugation and it results into π bond formation by p – p overlap.

Key **(C)**

49. Aqueous solution of $Na_2S_2O_3$ on reaction with Cl_2 gives

(A) $Na_2S_4O_6$ (B) $NaHSO_4$

(C) NaCl (D) NaOH

Sol $Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$

Key **(B)**

50. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of

(A) nitrogen (B) oxygen
(C) carbon dioxide (D) argon

Sol $Ag + NaCN + O_2 + H_2O \longrightarrow Na[Ag(CN)_2] + NaOH$

Key **(B)**

51. 2.5 mL of $\frac{2}{5}$ M weak monoacidic base ($K_b = 1 \times 10^{-12}$ at 25°C) is titrated with $\frac{2}{15}$ M HCl in water at 25°C. the concentration of H^+ at equivalence point is ($K_w = 1 \times 10^{-14}$ at 25°C)

(A) 3.7×10^{-13}M (B) 3.2×10^{-7}M
(C) 3.2×10^{-2} M (D) 2.7×10^{-2}M

Sol 0.4M BOH ($K_b = 10^{-1`2}$) + $\frac{2}{15}$MHCl

$pH = \frac{1}{2}[pK_w - pK_b - \log c]$

$= \frac{1}{2}[14 - 12 - \log 0.1]$

$= 1.5$, $[H^+] = 10^{-1.5} = 10^{0.5} \times 10^{-2}$

$= 3.2 \times 10^{-2}$M

Key **(C)**

52. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reaction is

(A) 0.5 mol^{-1} dm^3 (B) 1.0 mol dm^{-3}
(C) 1.5 mol dm^{-3} (D) 2.0 mol^{-1} dm^3

Sol $K_1 = \frac{0.693}{40}$

$K_0 = \frac{1.386}{2 \times 20} = \frac{1.386}{40} \min^{-1}$

$\frac{K_1}{K_0} = \frac{0.693}{1.386} = \frac{1}{2}$

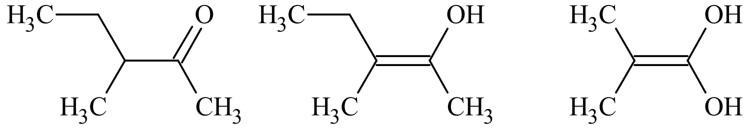
Key **(A)**

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

53. The correct statement(s) concerning the structures E, F and G is (are)



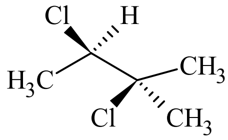
(E) (F) (G)

(A) E, F and G are resonance structures
(B) E, F and E, G are tautomers
(C) F and G are geometrical isomers
(D) F and G are diastereomeers

Sol F is the enol form of E and so is also G for E. F and G are also geometrical isomers.

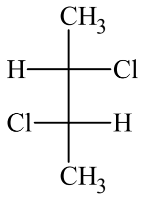
Key **(B), (C), (D)**

54. The correct statement(s) about the compound given below is (are)



(A) The compound is optically active
(B) The compound possesses centre of symmetry
(C) The compound possesses plane of symmetry
(D) The compound possesses axis of symmetry

Sol



Two chiral centres are of same configurations and hence optically active.

Key **(A)**

55. A gas described by van der Waals equation
(A) behaves similar to an ideal gas in the limit of large molar volumes
(B) behaves similar to an ideal gas in the limit of large pressure
(C) is characterized by van der Waals coefficients that are dependent on the identity of the gas but are independent of the temperature
(D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally

Sol $$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

When V is very large V – b $\approx$ V and $P+\frac{a}{V^2}\approx P,$ so PV = RT (for mole of an ideal gas). Due to inward pull acting on molecule striking the wall, the pressure decreases. The van der Waal's constants "a" and "b" are characteristics of a gas and as per van der Waal's they are temperature independent.

Key **(A), (C), (D)**

56. A solution of colourless salt H on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt(s) H is (are)
(A) NH_4NO_3
(B) NH_4NO_2
(C) NH_4Cl
(D) $(NH_4)_2SO_4$

Sol

$$NH_4X + NaOH \longrightarrow NaX + H_2O + NH_3\uparrow$$
$$(X^- = NO_3^-,\ NO_2^-,\ Cl^-,\ SO_4^{2-})$$
$$NO_3^- + 4Zn + 7OH^- \longrightarrow 4ZnO_2^{-2} + 2H_2O + NH_3\uparrow$$
$$NO_2^- + 3Zn + 5OH^- \longrightarrow 3ZnO_2^{-2} + H_2O + NH_3$$

Key **(A), (B)**

SECTION – III

Assertion – Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

57. STATEMENT-1: The plot of atomic number (y-axis versus number of neutrons (x-axis) for stable nuclei shows a curvature towards x-axis from theline of 45° slope as the atomic numbr is increased.

and

STATEMENT-2: Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons in heavier nuclides.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol (A) When Z > 20, the number of neutron must increase above the number of protons so as to overcome proton-proton repulsion i.e., $\frac{n}{p} > 1$.

Key **(A)**

58. STATEMENT-1: For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

and

STATEMENT-2: At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol G is minimum while $\Delta G = 0$ at equilibrium

Key **(D)**

59. STATEMENT-1: Bromobenzene upon reaction with Br_2/Fe gives 1, 4-dibromobenzeen as the major product.

and

STATEMENT-2: In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile..

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol In bromobenzene, inductive effect is responsible for deactivating the benzene nucleus and has no effect on directive influence. The directive influence is governed solely by mesomeeric effect.

Key **(C)**

60. STATEMENT-1: Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.

and

STATEMENT-2: The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to 'inert pair effect'.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol Sn and Pb both have ns^2np^2 configurations of their valence shells. Moving down a group, the ns electron pair becomes more and more inert towards bonding called inter pair effect. This is maximum in Pb. So Pb^{4+} tends to get reduced to Pb^{2+} i.e., Pb^{4+} is stronger oxidizing agent then Sn^{4+} in which the inert pair effect is relatively less.

Key **(C)**

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 61 to 63

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when solute molecules are added to get homogeneous solution. These are called colligative properties. Applications of colligative properties are very useful in day-to-day life. One of its examples is the use of ethylene glycol and water mixtures as anti-freezing liquid in the radiator of automobiles.

A solution **M** is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture is 0.9

Given: Freezing point depression constant of water $(K_f^{water}) = 1.86$ K kg mol^{-1}
Freezing point depression constant of ethanol $(K_f^{ethanol}) = 2.0$ K kg mol^{-1}
Boiling point elevation constant of water $(K_b^{water}) = 0.52$ K kg mol^{-1}
Boiling point elevation constant of ethanol $(K_b^{ethanol}) = 1.2$ K kg mol^{-1}
Standard freezing point of water = 273K
Standard freezing point of ethanol = 155.7 K
Standard boiling point of water = 373 K
Standard boiling point of ethanol = 351.5 K
Vapour pressure of pure water = 32.8 mm Hg
Vapour pressure of pure ethanol = 40 mm Hg
Molecular weight of water = 18 g mol^{-1}
Molecular weight of ethanol = 46 g mol^{-1}

In answering the following questions, consider the solutions to be ideal dilute solutions and slutes to be non-volatile and non-dissociateive.

61. The freezing point of the solution **M** is
(A) 268.7 K (B) 268.5 K
(C) 234.2 K (D) 150.9 K

Sol 0.1 mole water in 0.9 mol i.e. 0.9×46g ethanol

$\therefore$ molality of water in the solution = $\frac{0.1 \times 1000}{0.9 \times 46}$

$= \frac{100}{41} = 2.5$

$\Delta T_f = k_f \cdot C_m = 2 \times 2.5 = 5$

T_f of solution = 155.7 – 5 = 150.7K

Key **(D)**

62. The vapour pressure of the solution **M** is
(A) 39.3 mm Hg (B) 36.0 mm Hg
(C) 29.5 mm Hg (D) 28.8 mm Hg

Sol $p_{mix} = x_1 p_1^0 + x_2 p_2^0$
$= 0.1 \times 32{,}5 + 0.9 \times 40$
$= 3.28 + 36 = 39.28$ mm

Key **(A)**

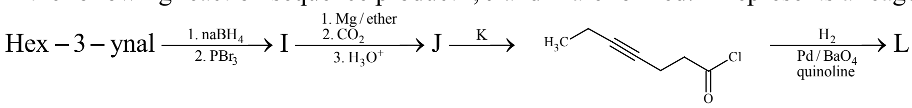
63. Water is added to the solution M such that the mole fraction of water in the solution becomes 0.9 The boiling point of this solution is
(A) 380.4 K (B) 376.2 K
(C) 373.5 K (D) 354.7 K

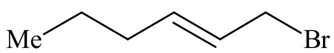
Sol $x_{C_2H_5OH} = 0.1,\ \ x_{H_2O} = 0.9$
18×0.9g of H_2O has 0.1 mol of C_2H_5OH
$\therefore$ molality of $C_2H_5OH = \dfrac{0.1}{18\times 0.9}\times 1000 = \dfrac{100}{16.2}$ mole KS^{-1}
$\therefore\ \Delta T_b = 0.52 \times \dfrac{100}{16.2} = \dfrac{52}{16.2} = 3.2$
$\therefore$ boiling point = 373 + 3.2 = 376.2K

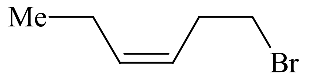
Key **(B)**

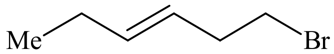
Paragraph for Questions Nos. 64 to 66

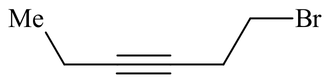
In the following reaction sequence product I, J and L are formed. K represents a reagent.

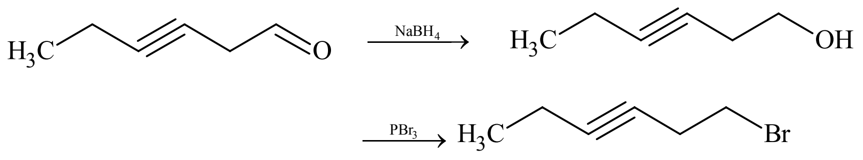


64. The structure of the product I is:
(A)


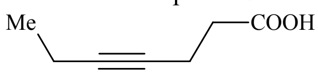
(B)


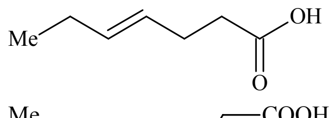
(C)


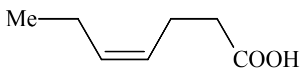
(D)


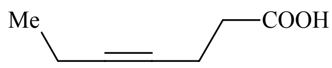
Sol


Key **(D)**

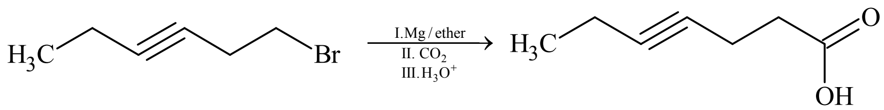
65. The structures of compound J and K, respectively, are
(A)


and $SOCl_2$
(B)


and SO_2Cl_2
(C)


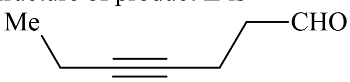
and $SOCl_2$
(D)


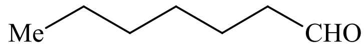
and CH_3SO_2Cl

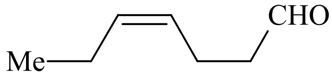
Sol


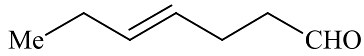
Key (A)

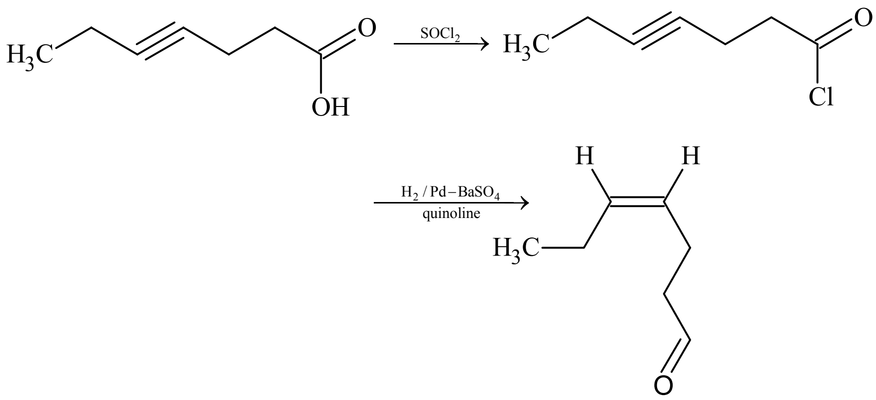
66. The structure of product **L** is

(A) 

(B) 

(C) 

(D) 

Sol 

Key (C)

Paragraph for Questions Nos. 67 to 69

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water. Nitrates re difficult to reduce under the laboratory conditions but microbes do it easily. Ammonia forms large number of complexes with transition metal ions. Hybridization easily explains the ease of sigma donation capability of NH_3 and PH_3. Phosphine is a flammable gas and is prepared from white phosphorous.

67. Among the following, the correct statement is
(A) Phosphates have no biological significance in humans
(B) Between nitrates and phosphates, phosphates are less abundant in earth's crust
(C) Between nitrates and phosphates, nitrates are less abundant in earth's crust
(D) Oxidation of nitrates is possible in soil.

Sol Nitrates being water soluble and as they get reduced by microbes so obviously its abundance will decrease.

Key (C)

68. Among the following, the correct statement is
(A) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional
(B) Between NH_3 and PH_3, PH_3 si a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(C) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(D) Between NH_3 and PH_3, PH_3 is a better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional.

Sol NH_3 is a stronger Lewis base than PH_3.

Key (C)

69. White phosphours on reaction with NaOH gives pH_3 as one of the products. This is a
(A) dimerization reaction
(B) disproportionation reaction
(D) condensation reaction
(D) precipitation reaction

Sol $\overset{0}{\underline{P}_4} + 3NaOH + 3H_2O \longrightarrow 3NaH_2\overset{+1}{\underline{P}}O_2 + \overset{-3}{\underline{P}}H_3$

Here P_4 is oxidized to NaH_2PO_2 and it is reduced to PH_3.

Key (B)